

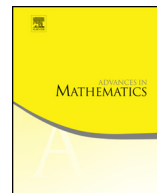


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Homological stability for automorphism groups

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ABSTRACT

Given a family of groups admitting a braided monoidal structure (satisfying mild assumptions) we construct a family of spaces on which the groups act and whose connectivity yields, via a classical argument of Quillen, homological stability for the family of groups. We show that stability also holds with both polynomial and abelian twisted coefficients, with no further assumptions. This new construction of a family of spaces from a family of groups recovers known spaces in the classical examples of stable families of groups, such as the symmetric groups, general linear groups and mapping class groups. By making systematic the proofs of classical stability results, we show that they all hold with the same type of coefficient systems, obtaining in particular without any further work new stability theorems with twisted coefficients for the symmetric groups, braid groups, automorphisms of free groups, unitary groups, mapping class groups of non-orientable surfaces and mapping class groups of 3-manifolds. Our construction can also be applied to families of groups not considered before in the context of homological stability. As a byproduct of our work, we construct the braided analogue of the category FI of finite sets and injections relevant to the present context, and define polynomiality for functors in the context of pre-braided monoidal categories.

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Contents

1.	Homogeneous and locally homogeneous categories	544
1.1.	(Locally) homogeneous categories from groupoids	548
1.2.	A graphical calculus for pre-braided categories	553
2.	The space of destabilisations associated to a pair (A, X)	554
2.1.	Simplicial complexes versus semi-simplicial sets	555
3.	Homological stability with constant and abelian coefficients	561
3.1.	Subgroups containing the commutator subgroup	570
3.2.	Stable homology	571
4.	Stability with twisted coefficients	572
4.1.	Coefficient systems	573
4.2.	Lower suspension, and suspension of coefficient systems	575
4.3.	Coefficient systems of G_∞^{ab} -modules	577
4.4.	Finite degree coefficient systems	579
4.5.	The stability theorem	582
4.6.	General properties of the relative homology groups	583
4.7.	Proof of the stability theorem (Theorem 4.20)	586
5.	Examples	594
5.1.	Symmetric groups	594
5.2.	Automorphisms of free groups and free products of groups	597
5.2.1.	Stability for $\text{Aut}(F_n)$	597
5.2.2.	More general free product stabilisation	599
5.3.	General linear groups	601
5.4.	Unitary groups	605
5.5.	Automorphism groups of direct products of groups	608
5.6.	Braid groups and mapping class groups of surfaces	609
5.6.1.	The groupoids of surfaces \mathcal{M}_2 , \mathcal{M}_2^- , and \mathcal{B}_2	610
5.6.2.	Braid groups and stabilisation by punctures	613
5.6.3.	Genus stabilisation for orientable surfaces	616
5.6.4.	Genus stabilisation for non-orientable surfaces	618
5.7.	Mapping class groups of 3-manifolds	620
	References	623

A family of groups

$$G_1 \hookrightarrow G_2 \hookrightarrow \cdots \hookrightarrow G_n \hookrightarrow \cdots$$

is said to satisfy *homological stability* if the induced maps

$$H_i(G_n) \longrightarrow H_i(G_{n+1})$$

are isomorphisms in a range $0 \leq i \leq f(n)$ increasing with n . In this paper, we prove that homological stability always holds if there is a monoidal category \mathcal{C} satisfying a certain hypothesis and a pair of objects A and X in \mathcal{C} , such that G_n is the group of automorphisms of $A \oplus X^{\oplus n}$ in \mathcal{C} . We show that stability holds not just for constant coefficients, but also for both *polynomial* and *abelian* coefficients, without any further assumption on \mathcal{C} .

The polynomial coefficient systems considered here are functors $F : \mathcal{C} \rightarrow \mathbb{Z}\text{-Mod}$ satisfying a finite degree condition. They are generalisations of polynomial functors

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