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Representation type for block algebras of Hecke algebras of classical type

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We find representation type of the cyclotomic quiver Hecke algebras $R^{\Lambda}(\beta)$ of level two in affine type A. In particular, we have determined representation type for all the block algebras of Hecke algebras of classical type (except for characteristic two in type D), which has not been known for a long time. As an application of this result, we prove that block algebras of finite representation type are Brauer tree algebras whose Brauer trees are straight lines without exceptional vertex if the Hecke algebras are of classical type in good characteristic. We conjecture that this statement should hold for Hecke algebras of exceptional type.

0. Introduction

Hecke algebras of classical type are important in modular representation theory of finite groups of Lie type in non-defining characteristic. For example, those of type A and B appear as endomorphism algebras in Harish-Chandra series. Because of the impor-

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tance, they have received detailed studies since 1980's when Dipper and James initiated the field.

A long standing open problem in modular representation theory of Hecke algebras was to determine representation type of Hecke algebras of type B with equal or unequal parameters blockwise. For Hecke algebras of type A, representation type of their block algebras was determined by Erdmann and Nakano [16].[2] Another proof is given by the author, Iijima and Park [5]. In that paper, we have determined representation type for a one parameter family of finite quiver Hecke algebras in affine type A, and the family includes block algebras of Hecke algebras in type A at a special parameter value.[3] On the other hand, we may embed Hecke algebras of type D to Hecke algebras of type B. Assuming that the base field has an odd characteristic, we may apply Clifford theory to determine representation type of block algebras of Hecke algebras in type D. See Appendix 2. Therefore, the problem to solve is to determine representation type of block algebras of Hecke algebras in type B.

Let $\mathbf{k}$ be an algebraically closed field, $q \neq 1, Q \in \mathbf{k}^\times$. The Hecke algebra of type B is the $\mathbf{k}$-algebra $\mathcal{H}_n(q,Q)$ defined by generators $T_0, T_1, \ldots, T_{n-1}$ and relations

$$(T_0 - Q)(T_0 + 1) = 0, \quad (T_i - q)(T_i + 1) = 0 \ \ (1 \le i < n)$$

$$T_0T_1T_0T_1 = T_1T_0T_1T_0, \quad T_iT_{i+1}T_i = T_{i+1}T_iT_{i+1} \ \ (1 \le i < n-1)$$

$$T_iT_j = T_jT_i \ \ (j \neq i \pm 1).$$

We have two cases to consider. When $-Q \notin q^{\mathbb{Z}}$, then Morita equivalence theorem by Dipper and James [14] implies that we determine representation type of an (outer) tensor product of two block algebras of Hecke algebras of type A. Then, it suffices to consider tensor product of block algebras of finite or tame representation type, as representation types for other tensor products are clear. Block algebras of finite representation type are Morita equivalent to the principal block of rank $\ell + 1$ [28]. Further, block algebras of tame representation type appear only when $\ell = 1$ [16], and they are Morita equivalent to either the principal block of rank 4 or 5 by Scopes' Morita equivalence. Therefore, we can determine the representation type blockwise when $-Q \notin q^{\mathbb{Z}}$ by studying those rather simple tensor product algebras. See Appendix 1. Hence we focus on the case $-Q = q^s$, for some $s \in \mathbb{Z}$, and the Hecke algebra of type B in this case is isomorphic to the cyclotomic Hecke algebra associated with level two dominant weight $\Lambda = \Lambda_0 + \Lambda_s$.

[2] The proof in [16] uses a result which judged representation type by complexity, but this result is known to be false. However, the authors had a different proof and they were able to avoid its use in the second proof. Thus, their final result in [16] holds without any change. See explanation in the corrigendum to [4].

[3] Warning for those who are not familiar with Hecke algebras. The parameter q which appears in the definition of the Hecke algebra of type A is fixed and it determines the affine Lie type $A_\ell^{(1)}$. The one parameter we mention here is another parameter which comes from polynomials $Q_{ij}(u,v)$ in the definition of the finite quiver Hecke algebra in affine type A.