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Diffusive wave in the low Mach limit for compressible Navier–Stokes equations

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ABSTRACT

The low Mach limit for 1D non-isentropic compressible Navier–Stokes flow, whose density and temperature have different asymptotic states at infinity, is rigorously justified. The problems are considered on both well-prepared and ill-prepared data. For the well-prepared data, the solutions of compressible Navier–Stokes equations are shown to converge to a nonlinear diffusion wave solution globally in time as Mach number goes to zero when the difference between the states at $\pm\infty$ is suitably small. In particular, the velocity of diffusion wave is only driven by the variation of temperature. It is further shown that the solution of compressible Navier–Stokes system also has the same property when Mach number is small, which has never been observed before. The convergence rates on both Mach number and time are also obtained for the well-prepared data. For the ill-prepared data, the limit relies on the uniform estimates including weighted time derivatives and an extended convergence lemma. And the difference between the states at $\pm\infty$ can be arbitrary large in this case.

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1. Introduction

The non-isentropic Navier–Stokes system in $\mathbb{R}^n$ is as follows

$$\begin{cases} \partial_t\rho + \mathrm{div}(\rho u) = 0, \\ \partial_t(\rho u) + \mathrm{div}(\rho u\otimes u) + \nabla P = \mathrm{div}(2\mu D(u)) + \nabla(\lambda \mathrm{div} u), \\ \partial_t(\rho(e+\frac{1}{2}|u|^2)) + \mathrm{div}(\rho u(e+\frac{1}{2}|u|^2) + Pu) = \mathrm{div}(\kappa\nabla\mathcal{T}) + \mathrm{div}(2\mu D(u)u + \lambda \mathrm{div} u u), \end{cases} \tag{1.1}$$

for $t>0, x\in\mathbb{R}^n$. Here the unknown functions ρ, u, and $\mathcal{T}$ represent the density, velocity, and temperature, respectively. The pressure function and internal function are defined by

$$P = R\rho\mathcal{T}, \quad e = c_v\mathcal{T}, \tag{1.2}$$

where the parameters $R>0$ and $c_v>0$ are the gas constant and the heat capacity at constant volume, respectively. $D(u)$ is the deformation tensor given by

$$D(u) = \frac{1}{2}(\nabla u + (\nabla u)^t),$$

where $(\nabla u)^t$ denote the transpose of matrix ∇u. μ and λ are the Lamé viscosity coefficients which satisfy

$$\mu>0, \quad 2\mu + n\lambda > 0,$$

and $\kappa>0$ is the heat conductivity coefficient. For simplicity, we assume that μ, λ, κ are constants.

It is noted $c_v = (\gamma-1)/R$ where $\gamma>1$ is the adiabatic exponent. In this paper, since we consider the low Mach limit for smooth solutions of non-isentropic compressible Navier–Stokes equation, we normalize $R=1$ and $c_v=1$ for simplicity of presentation, see also [25]. And we point out that our results and energy estimates in this paper still