

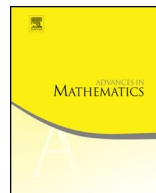


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Sharp Hardy–Adams inequalities for bi-Laplacian on hyperbolic space of dimension four [☆]

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ABSTRACT

We establish sharp Hardy–Adams inequalities on hyperbolic space $\mathbb{B}^4$ of dimension four. Namely, we will show that for any $\alpha > 0$ there exists a constant $C_\alpha > 0$ such that

$$\int_{\mathbb{B}^4} (e^{32\pi^2 u^2} - 1 - 32\pi^2 u^2) dV = 16 \int_{\mathbb{B}^4} \frac{e^{32\pi^2 u^2} - 1 - 32\pi^2 u^2}{(1 - |x|^2)^4} dx \leq C_\alpha$$

for any $u \in C_0^\infty(\mathbb{B}^4)$ with

$$\int_{\mathbb{B}^4} \left(-\Delta_{\mathbb{H}} - \frac{9}{4} \right) (-\Delta_{\mathbb{H}} + \alpha) u \cdot u dV \leq 1.$$

As applications, we obtain a sharpened Adams inequality on hyperbolic space $\mathbb{B}^4$ and an inequality which improves the classical Adams' inequality and the Hardy inequality simultaneously. The later inequality is in the spirit of the Hardy–Trudinger–Moser inequality on a disk in dimension two given by Wang and Ye in [46] and on any convex planar domain by Lu and Yang in [33].

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The Fourier analysis techniques on hyperbolic and symmetric spaces play an important role in our work.

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1. Introduction

Our main purpose of this article is to establish sharp Hardy–Adams inequalities on $\mathbb{B}^4$, a hyperbolic space in dimension four.

We first recall the classical Trudinger–Moser inequality in any finite domain of Euclidean spaces. Let $\Omega \subset \mathbb{R}^n (n \geq 2)$ be a bounded domain and $1 \leq q \leq \frac{np}{n-kp}$. Then it is well known that the Sobolev embedding theorem tells us the embedding $W_0^{k,p}(\Omega) \subset L^q(\Omega)$ is continuous when $kp < n$. However, in general $W_0^{1,n}(\Omega) \not\subset L^\infty(\Omega)$. Trudinger [45] established in the borderline case that $W_0^{1,n}(\Omega) \subset L_{\varphi_n}(\Omega)$, where $L_{\varphi_n}(\Omega)$ is the Orlicz space associated with the Young function $\varphi_n(t) = \exp(\beta|t|^{n/n-1}) - 1$ for some $\beta > 0$ (see also Yudovich [49], Pohozaev [43]). In 1971, Moser sharpened the Trudinger inequality in [39] by finding the optimal β :

Theorem 1.1 (Trudinger–Moser). *Let Ω be a domain with finite measure in Euclidean n -space $\mathbb{R}^n$, $n \geq 2$. Then there exists a sharp constant $\beta_n = n \left(\frac{n\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2}+1)} \right)^{\frac{1}{n-1}}$ such that*

$$\frac{1}{|\Omega|} \int_{\Omega} \exp(\beta|u|^{\frac{n}{n-1}}) dx \leq c_0$$

for any $\beta \leq \beta_n$, any $u \in W_0^{1,n}(\Omega)$ with $\int_{\Omega} |\nabla u|^n dx \leq 1$. This constant β_n is sharp in the sense that if $\beta > \beta_n$, then the above inequality can no longer hold with some c_0 independent of u .

In 1988, D. Adams extended such an inequality to higher order Sobolev spaces. In fact, Adams proved the following:

Theorem 1.2. *Let Ω be a domain in $\mathbb{R}^n$ with finite n -measure and m be a positive integer less than n . There is a constant $c_0 = c_0(m, n)$ such that for all $u \in C^m(\mathbb{R}^n)$ with support contained in Ω and $\|\nabla^m u\|_{n/m} \leq 1$, the following uniform inequality holds*

$$\frac{1}{|\Omega|} \int_{\Omega} \exp(\beta_0(m, n)|u|^{n/(n-m)}) dx \leq c_0, \quad (1.1)$$

where

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