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Pointwise Hölder exponents of the complex analogues of the Takagi function in random complex dynamics

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ABSTRACT

We consider hyperbolic random complex dynamical systems on the Riemann sphere with separating condition and multiple minimal sets. We investigate the Hölder regularity of the function T of the probability of tending to one minimal set, the partial derivatives of T with respect to the probability parameters, which can be regarded as complex analogues of the Takagi function, and the higher partial derivatives C of T. Our main result gives a dynamical description of the pointwise Hölder exponents of T and C, which allows us to determine the spectrum of pointwise Hölder exponents by employing the multifractal formalism in ergodic theory. Also, we prove that the bottom of the spectrum α_- is strictly less than 1, which allows us to show that the averaged system acts chaotically on the Banach space C^α of α-Hölder continuous functions for

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every $\alpha \in (\alpha_-, 1)$, though the averaged system behaves very mildly (e.g. we have spectral gaps) on C^β for small $\beta > 0$.

1. Introduction and statement of results

In this paper, we consider random dynamical systems of rational maps on the Riemann sphere $\widehat{\mathbb{C}}$. The study of random complex dynamics was initiated by J.E. Fornaess and N. Sibony ([3]). There are many new interesting phenomena in random dynamical systems, so called randomness-induced phenomena or noise-induced phenomena, which cannot hold in the deterministic iteration dynamics. For the motivations and recent research of random complex dynamical systems focused on the randomness-induced phenomena, see the second author's works [22,24–26]. In these papers it was shown that for a generic i.i.d. random dynamical system of complex polynomials of degree two or more, the system acts very mildly on the space of continuous functions on $\widehat{\mathbb{C}}$ and on the space $C^\alpha(\widehat{\mathbb{C}})$ for small $\alpha \in (0,1)$, where $C^\alpha(\widehat{\mathbb{C}})$ denotes the Banach space of α-Hölder continuous functions on $\widehat{\mathbb{C}}$ endowed with α-Hölder norm, but under certain conditions the system still acts chaotically on the space $C^\beta(\widehat{\mathbb{C}})$ for some $\beta \in (0,1)$ close to 1. Thus, we investigate the gradation between chaos and order in random (complex) dynamical systems.

In order to show the main ideas of the paper, let Rat denote the set of all non-constant rational maps on $\widehat{\mathbb{C}}$. This is a semigroup whose semigroup operation is the composition of maps. Throughout the paper, let $s \geq 1$ and let $(f_1, \dots, f_{s+1}) \in (\mathrm{Rat})^{s+1}$ with $\deg(f_i) \geq 2$, $i = 1, \dots, s+1$. Let $\mathbf{p} = (p_1, \dots, p_s) \in (0,1)^s$ with $\sum_{i=1}^s p_i < 1$ and let $p_{s+1} := 1 - \sum_{i=1}^s p_i$. We consider the (i.i.d.) random dynamical system on $\widehat{\mathbb{C}}$ such that at every step we choose f_i with probability p_i. This defines a Markov chain with state space $\widehat{\mathbb{C}}$ such that for each $x \in \widehat{\mathbb{C}}$ and for each Borel measurable subset A of $\widehat{\mathbb{C}}$, the transition probability $p(x, A)$ from x to A is equal to $\sum_{i=1}^{s+1} p_i 1_A(f_i(x))$, where 1_A denotes the characteristic function of A. Let $G = \langle f_1, \dots, f_s, f_{s+1}\rangle$ be the rational semigroup (i.e., subsemigroup of Rat) generated by $\{f_1, \dots, f_{s+1}\}$. More precisely, $G = \{f_{\omega_n} \circ \cdots \circ f_{\omega_1} : n \in \mathbb{N}, \omega_1, \dots, \omega_n \in \{1, \dots, s+1\}\}$. We denote by $F(G)$ the maximal open subset of $\widehat{\mathbb{C}}$ on which G is equicontinuous with respect to the spherical distance on $\widehat{\mathbb{C}}$. The set $F(G)$ is called the Fatou set of G, and the set $J(G) := \widehat{\mathbb{C}} \setminus F(G)$ is called the Julia set of G. We remark that in order to investigate random complex dynamical systems, it is very important to investigate the dynamics of associated rational semigroups. The first study of dynamics of rational semigroups was conducted by A. Hinkkanen and G.J. Martin ([5]), who were interested in the role of polynomial semigroups (i.e., semigroups of non-constant polynomial maps) while studying various one-complex-dimensional moduli spaces for discrete groups, and by F. Ren's group ([4]), who studied such semigroups from the perspective of random dynamical systems. For the interplay of random complex dynamics and dynamics of rational semigroups, see [17–26,14,27,28,6,7].