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Khovanov homology and the symmetry group of a knot

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To my grandfather, Karl Erik Snider

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ABSTRACT

We introduce an invariant of tangles in Khovanov homology by considering a natural inverse system of Khovanov homology groups. As application, we derive an invariant of strongly invertible knots; this invariant takes the form of a graded vector space that vanishes if and only if the strongly invertible knot is trivial. While closely tied to Khovanov homology — and hence the Jones polynomial — we observe that this new invariant detects non-amphicheirality in subtle cases where Khovanov homology fails to do so. In fact, we exhibit examples of knots that are not distinguished by Khovanov homology but, owing to the presence of a strong inversion, may be distinguished using our invariant. This work suggests a strengthened relationship between Khovanov homology and Heegaard Floer homology by way of two-fold branched covers that we formulate in a series of conjectures.

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The reduced Khovanov homology of an oriented link L in the three-sphere is a bi-graded vector space $\widetilde{\mathrm{Kh}}(L)$ for which the graded Euler characteristic $\sum_{u,q}(-1)^u t^q \dim \widetilde{\mathrm{Kh}}^u_q(L)$ recovers the Jones polynomial of L [16,18]. This homological link invariant is known to detect the trivial knot. Precisely, Kronheimer and Mrowka prove that $\dim \widetilde{\mathrm{Kh}}(K) = 1$ if and only if K is the trivial knot [21]. It remains an open problem to determine if the analogous detection result holds for the Jones polynomial.

Preceding the work of Kronheimer and Mrowka are a range of applications of Khovanov homology in low-dimensional topology. Perhaps most recognized among these is Rasmussen's combinatorial proof of the Milnor conjecture by way of the s invariant [37]. Other examples include Ng's bound on the Thurston–Bennequin number [28], Plamenevskaya's invariant of transverse knots [35], and obstructions to finite fillings on strongly invertible knots due to the author [47,48].

There are two features common to each of these applications. First, the quantity extracted from Khovanov homology is an integer (a particular grading [28,35,37], a count of a collection of gradings [47], or a dimension count [21]); and second, the quantity measured is not one that can be extracted from the Jones polynomial — additional structure in Khovanov homology is essential in each case. The latter points to a clear advantage of Khovanov homology over the Jones polynomial, while the former suggests that further applications might be possible by considering more of the available structure.

This paper is principally concerned with developing new applications of the graded information in Khovanov homology.

Tangle invariants in Khovanov homology

As with the Jones polynomial, tangle decompositions provide an approach to calculation and an enrichment of structure in Khovanov homology. For example, Bar-Natan's work [5] gave rise to a considerable improvements in calculation speed [6]. Bar-Natan works in a category of formal complexes of tangles up to homotopy (modulo certain topological relations). On the other hand, Khovanov defines an algebraic invariant that is more natural in certain settings [17] — particularly in relation to two-fold branched covers and bordered Floer homology [2]. There are a range of other generalized tangle invariants in Khovanov homology [1,9,22,39,40] and the state of the art is nicely summarized by Roberts [40].

We introduce a new tangle invariant in Khovanov homology that is perhaps best aligned with the work of Grigsby and Wehrli [9]. The tangles considered are pairs $T = (B^3, \tau)$, where τ is a pair of properly embedded disjoint smooth arcs (together with a potentially empty collection of embedded disjoint closed components). These tangles will be endowed with a sutured structure (see Definition 3, and compare the definitions of [9, Section 5]), which may be thought of as a partition of the four points of $\partial\tau \subset \partial B^3$ into two pairs of points. Namely, we replace B^3 with the product $D^2 \times I$ and distinguish the annular subset of the boundary $\partial D^2 \times I$ as the suture (see Fig. 1). Equivalence of