

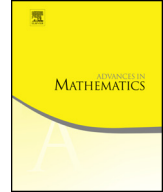


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Bloch functions and asymptotic tail variance [☆]



Haakan Hedenmalm ^{a,b,*}

^a Department of Mathematics, KTH Royal Institute of Technology, S-10044 Stockholm, Sweden

^b Department of Mathematics and Mechanics, Saint Petersburg State University, 28 Universitetski pr., St-Petersburg 198504, Russia

ARTICLE INFO

Article history:

Received 24 March 2017

Accepted 11 April 2017

Communicated by Charles Fefferman

MSC:

primary 30H30, 32A25

secondary 30C62, 47B38, 37F30,

32A40

Keywords:

Asymptotic variance

Asymptotic tail variance

Bloch function

Bergman projection

Quasiconformal

Holomorphic motion

ABSTRACT

Let \mathbf{P} denote the Bergman projection on the unit disk \mathbb{D} ,

$$\mathbf{P}\mu(z) := \int_{\mathbb{D}} \frac{\mu(w)}{(1-z\bar{w})^2} dA(w), \quad z \in \mathbb{D},$$

where dA is normalized area measure. We prove that if $|\mu(z)| \leq 1$ on \mathbb{D} , then the integral

$$I_{\mu}(a, r) := \int_0^{2\pi} \exp \left\{ a \frac{r^4 |\mathbf{P}\mu(re^{i\theta})|^2}{\log \frac{1}{1-r^2}} \right\} \frac{d\theta}{2\pi}, \quad 0 < r < 1,$$

has the bound $I_{\mu}(a, r) \leq C(a) := 10(1-a)^{-3/2}$ for $0 < a < 1$, irrespective of the choice of the function μ . Moreover, for $a > 1$, no such uniform bound is possible. We interpret the theorem in terms the *asymptotic tail variance* of such a Bergman projection $\mathbf{P}\mu$ (by the way, the asymptotic tail variance induces a seminorm on the Bloch space). This improves upon earlier work of Makarov, which covers the range $0 < a < \frac{\pi^2}{64} = 0.1542\dots$. We then apply the theorem to obtain an estimate of the universal integral means spectrum for conformal mappings with a k -quasiconformal extension, for $0 < k < 1$. The estimate reads, for $t \in \mathbb{C}$ and $0 < k < 1$,

[☆] This research was supported by RNF grant 14-41-00010.

* Correspondence to: Department of Mathematics, KTH Royal Institute of Technology, S-10044 Stockholm, Sweden.

E-mail address: haakanh@math.kth.se.

$$B(k, t) \leq \begin{cases} \frac{1}{4}k^2|t|^2(1+7k)^2, & \text{for } |t| \leq \frac{2}{k(1+7k)^2}, \\ k|t| - \frac{1}{(1+7k)^2}, & \text{for } |t| \geq \frac{2}{k(1+7k)^2}, \end{cases}$$

which should be compared with the conjecture by Prause and Smirnov to the effect that for real t with $|t| \leq 2/k$, we should have $B(k, t) = \frac{1}{4}k^2t^2$.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

1.1. Basic notation

We write \mathbb{R} for the real line, $\mathbb{R}_+ :=]0, +\infty[$ for the positive semi-axis, and \mathbb{C} for the complex plane. Moreover, we write $\mathbb{C}_\infty := \mathbb{C} \cup \{\infty\}$ for the extended complex plane (the Riemann sphere). For a complex variable $z = x + iy \in \mathbb{C}$, let

$$ds(z) := \frac{|dz|}{2\pi}, \quad dA(z) := \frac{dx dy}{\pi},$$

denote the normalized arc length and area measures as indicated. Moreover, we shall write

$$\Delta_z := \frac{1}{4} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right)$$

for the normalized Laplacian, and

$$\partial_z := \frac{1}{2} \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right), \quad \bar{\partial}_z := \frac{1}{2} \left(\frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right),$$

for the standard complex derivatives; then Δ factors as $\Delta_z = \partial_z \bar{\partial}_z$. Often we will drop the subscript for these differential operators when it is obvious from the context with respect to which variable they apply. We let \mathbb{C} denote the complex plane, \mathbb{D} the open unit disk, $\mathbb{T} := \partial\mathbb{D}$ the unit circle, and \mathbb{D}_e the exterior disk:

$$\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}, \quad \mathbb{D}_e := \{z \in \mathbb{C}_\infty : |z| > 1\}.$$

More generally, we write

$$\mathbb{D}(z_0, r) := \{z \in \mathbb{C} : |z - z_0| < r\}$$

for the open disk of radius r centered at z_0 .

Download English Version:

<https://daneshyari.com/en/article/5778469>

Download Persian Version:

<https://daneshyari.com/article/5778469>

[Daneshyari.com](https://daneshyari.com)