

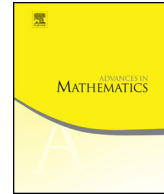


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Brill–Noether varieties of k -gonal curves

Nathan Pflueger

Department of Mathematics, Brown University, Box 1917, Providence, RI 02912,
United States

ARTICLE INFO

Article history:

Received 30 March 2016

Received in revised form 15 January 2017

Accepted 30 January 2017

Available online xxxx

Communicated by Ravi Vakil

Keywords:

Brill–Noether theory

Special divisors

Gonality

Tropical geometry

Chain of loops

ABSTRACT

We consider a general curve of fixed gonality k and genus g . We propose an estimate $\bar{\rho}_{g,k}(d, r)$ for the dimension of the variety $W_d^r(C)$ of special linear series on C , by solving an analogous problem in tropical geometry. Using work of Coppens and Martens, we prove that this estimate is exactly correct if $k \geq \frac{1}{5}g + 2$, and is an upper bound in all other cases. We also completely characterize the cases in which $W_d^r(C)$ has the same dimension as for a general curve of genus g .

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

We are concerned in this paper with the varieties $W_d^r(C)$ of special linear series on a general curve of fixed genus g and gonality k over an algebraically closed field K with mild restrictions on the characteristic. See [Situation 1.4](#) for the necessary hypotheses.

For a general curve C of genus g , we have $k = \lfloor \frac{g+3}{2} \rfloor$ and the Brill–Noether theorem [\[6\]](#) says that $\dim W_d^r(C)$ is equal to the Brill–Noether number

E-mail address: pflueger@math.brown.edu.

$$\rho_g(d, r) = g - (r + 1)(g - d + r),$$

unless $\rho_g(d, r) < 0$, in which case $W_d^r(C) = \emptyset$.

Our objective is to compute $\dim W_d^r(C)$ for non-generic values of k . We propose a modification $\bar{\rho}_{g,k}(d, r)$ of the Brill–Noether number, incorporating the value of k , and prove the following results. By convention, a negative estimate on $\dim W_d^r(C)$ means that $W_d^r(C)$ is empty.

Theorem 1.1. In [Situation 1.4](#), $\dim W_d^r(C) \leq \bar{\rho}_{g,k}(d, r)$.

In characteristic 0, we may combine [Theorem 1.1](#) with a lower bound from results of Coppens and Martens [\[4\]](#), to obtain the following sharp results.

Theorem 1.2. In [Situation 1.4](#), if $\text{char } K = 0$ and either $k \leq 5$ or $k \geq \frac{1}{5}g + 2$, then $\dim W_d^r(C) = \bar{\rho}_{g,k}(d, r)$.

Theorem 1.3. In [Situation 1.4](#), if $\text{char } K = 0$ and $\rho_g(d, r) \geq 0$, then $\dim W_d^r(C) = \rho_g(d, r)$ if and only if $r = 0$, $g - d + r = 1$, or $g - k \leq d - 2r$.

All three theorems use the following notation and hypotheses.

Situation 1.4. Fix nonnegative integers g, k, r, d such that $2 \leq k \leq \frac{g+3}{2}$ and $g - d + r > 0$. Let K be an algebraically closed field, and C be a general k -gonal curve of genus g over K . Also assume that

- if k is odd, then $\text{char } K \neq 2$,
- if $k = 4$ or 10 , then $\text{char } K \neq 3$, and
- if $k = 6$, then $\text{char } K \neq 5$.

The hypothesis $g - d + r > 0$ is harmless, since $g - d + r \leq 0$ would imply automatically that $W_d^r(C) = \text{Pic}^d(C)$. The peculiar restrictions on the characteristic of K arise in our proof when we must construct a *tame* morphism of metrized complexes with certain properties. The characteristic 0 assumption in [Theorems 1.2 and 1.3](#) is included because these make use of constructions from [\[3\]](#) and [\[4\]](#), which assume this hypothesis.

1.1. The estimate $\bar{\rho}_{g,k}(d, r)$

The estimate $\bar{\rho}_{g,k}(d, r)$ we refer to above is the following.

Definition 1.5. Let r' denote the minimum of r and $g - d + r - 1$. Define

$$\bar{\rho}_{g,k}(d, r) = \max_{\ell \in \{0, 1, 2, \dots, r'\}} (\rho_g(d, r - \ell) - \ell k).$$

Download English Version:

<https://daneshyari.com/en/article/5778477>

Download Persian Version:

<https://daneshyari.com/article/5778477>

[Daneshyari.com](https://daneshyari.com)