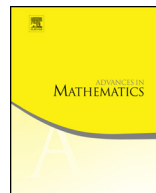




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Finite topology minimal surfaces in homogeneous three-manifolds



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ABSTRACT

We prove that any complete, embedded minimal surface M with finite topology in a homogeneous three-manifold N has positive injectivity radius. When one relaxes the condition that N be homogeneous to that of being locally homogeneous, then we show that the closure of M has the structure of a minimal lamination of N . As an application of this general result we prove that any complete, embedded minimal surface with finite genus and a countable number of ends is compact when the ambient space is S^3 equipped with a homogeneous metric of nonnegative scalar curvature.

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1. Introduction

In this paper we apply the Local Picture Theorem on the Scale of Topology, which is Theorem 1.1 in [7] (see Theorem 1.5 below for the statement of this result in the finite genus setting), to prove that the injectivity radii of certain minimal surfaces in certain Riemannian three-manifolds are never zero.

Theorem 1.1. *Let N be a complete, locally homogeneous³ three-manifold with positive injectivity radius. Then, every complete, embedded minimal surface of finite topology in N has positive injectivity radius.*

In the case that the ambient three-manifold N is isometric to $\mathbb{S}^2 \times \mathbb{R}$ with a scaling of its standard product metric, then this result follows from Theorem 15 in [12] where Meeks and Rosenberg also applied Theorem 1.1 in [7] to prove the stronger property that such minimal surfaces have bounded second fundamental form, linear ambient area growth and so they are also proper in $\mathbb{S}^2 \times \mathbb{R}$. For background material on the geometry and classification of homogeneous three-manifolds, see [6].

The next result removes the positive injectivity radius assumption for the ambient space N . The conclusion that we obtain in this setting is also weaker than the one in Theorem 1.1, as follows from the Minimal Lamination Closure Theorem in [12].

Corollary 1.2. *If M is a complete embedded minimal surface of finite topology in a complete, locally homogeneous three-manifold N , then the closure $\overline{M}$ has the structure of a minimal lamination of N . Furthermore:*

1. *Each limit leaf of $\overline{M}$ is stable (more precisely, the two-sided cover of the leaf is stable).*
2. *If N has positive scalar curvature, then M is proper in N .*
3. *If N is simply connected and has nonnegative scalar curvature, then M is proper in N .*
4. *If N is the round three-sphere $\mathbb{S}^3$, then M is compact.*

Remark 1.3. Item 1 of Corollary 1.2 still holds without the hypothesis on N to be locally homogeneous. On the other hand, it can be shown that there exists a Riemannian metric of positive scalar curvature on the three-sphere that admits a complete embedded minimal plane whose closure does not admit the structure of a minimal lamination (see e.g., [1]). Hence, our hypothesis that N is locally homogeneous is necessary for items 2, 3 of Corollary 1.2 to hold.

³ A Riemannian manifold N is *locally homogeneous* if given two points in N , balls of the same sufficiently small radius centered at these points are isometric.

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