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On random linear dynamical systems in a Banach space. I. Multiplicative Ergodic Theorem and Krein–Rutman type Theorems ☆

Zeng Lian [a,*], Yi Wang [b]

[a] College of Mathematics, Sichuan University, Chengdu, Sichuan, 610065, People's Republic of China

[b] Wu Wen-Tsun Key Laboratory, School of Mathematical Science, University of Science and Technology of China, Hefei, Anhui, 230026, People's Republic of China

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ABSTRACT

For linear random dynamical systems in a separable Banach space X, we derived a series of Krein–Rutman type Theorems with respect to co-invariant cone family with rank-k, which present a (quasi)-equivalence relation between the measurably co-invariant cone family and the measurably dominated splitting of X. Moreover, such (quasi)-equivalence relation turns out to be an equivalence relation whenever (i) $k = 1$; or (ii) in the frame of the Multiplicative Ergodic Theorem with certain Lyapunov exponent being greater than the negative infinity. For the second case, we thoroughly investigated the relations between the Lyapunov exponents, the co-invariant cone family and the measurably dominated splitting for linear random dynamical systems in X.

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* Corresponding author.
E-mail addresses: zenglian@gmail.com (Z. Lian), wangyi@ustc.edu.cn (Y. Wang).

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1. Introduction

Lyapunov exponents [21] play important roles in the study of the behavior of dynamical systems. One of the most celebrated theorems on Lyapunov exponents is the so-called the Multiplicative Ergodic Theorem, which was first obtained by Oseledets [32] in 1968. This theorem plays a crucial role in the Pesin theory for describing the dynamics of nonuniformly hyperbolic diffeomorphisms on compact manifolds. During recent decades, many remarkable works and extensions of Multiplicative Ergodic Theorem have been carried out from finite-dimensional systems to infinite-dimensional systems, and from deterministic dynamical systems to random dynamical systems (see [1,18,24,38,43] and the references therein).

In a separable Banach space X, the Multiplicative Ergodic Theorem proposes an effective approach for decoupling X into finitely (or infinitely)-many random co-invariant subspaces which are strongly measurable and the "angles" between them are tempered. The existence of Lyapunov exponents with different values gives rise to exponential dichotomies, that is a well-known "spectral-gap" condition (see e.g., [5,6,33]), between the various invariant subspaces. Such gap has been attracting interest broadly in deterministic systems for a long time because of its close relationship with the theory of invariant manifolds.

As was pointed out in [23,20], a genuinely essential condition for the existence of invariant manifolds is the existence of co-invariant cone family. Compared with the spectral gap condition, the invariant cones condition turns out to be more intuitive from the viewpoint of geometric description. While for the differential equations in question (see e.g., [3,8,23,41]), the cones condition can be verified by showing, roughly speaking, that the vector field on the boundaries of the cones point inward.

We will show in Section 6.1 that a family of finitely (or infinitely)-many nested invariant cones can be constructed under the assumptions of Multiplicative Ergodic Theorem in a separable Banach space X (see Lian and Lu [18]). To be more precisely, let us consider the linear cocycle over the metric dynamical system $(\Omega, \mathcal{F}, \mathbb{P}, \theta)$ generated by a strongly measurable random variable $A : \Omega \to L(X)$,

$$T^n(\omega) := A(\theta^{n-1}\omega) \cdots A(\omega),\ n \in \mathbb{N}, \tag{1}$$

where $L(X)$ is the space of bounded linear operators from X to itself; and moreover, we assume that $A(\omega)$ is injective almost everywhere. Define that

$$\lambda_0(\omega) = \lim_{n\to\infty} \frac{1}{n} \log \|T^n(\omega)\|, \tag{2}$$

$$\kappa(\omega) = \lim_{n\to\infty} \frac{1}{n} \log \|T^n(\omega)\|_\kappa, \tag{3}$$