Advances in Mathematics 312 (2017) 547–587

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Energy gap for Yang–Mills connections, II: Arbitrary closed Riemannian manifolds ☆

Paul M.N. Feehan

Department of Mathematics, Rutgers, The State University of New Jersey, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019, United States

ARTICLE INFO

Article history:
Received 4 May 2016
Received in revised form 2 March 2017
Accepted 22 March 2017
Available online 7 April 2017
Communicated by the Managing Editors

MSC:
primary 58E15, 57R57
secondary 37D15, 58D27, 70S15, 81T13

Keywords:
Energy gaps
Flat connections
Flat bundles
Gauge theory
Łojasiewicz–Simon gradient inequality
Morse theory on Banach manifolds
Closed Riemannian manifolds
Yang–Mills connections

ABSTRACT

We prove an $L^{d/2}$ energy gap result for Yang–Mills connections on principal G-bundles, P, over arbitrary, closed, Riemannian, smooth manifolds of dimension $d \geq 2$. We apply our version of the Łojasiewicz–Simon gradient inequality [16,19] to remove a positivity constraint on a combination of the Ricci and Riemannian curvatures in a previous $L^{d/2}$-energy gap result due to Gerhardt [23, Theorem 1.2] and a previous L^∞-energy gap result due to Bourguignon, Lawson, and Simons [10, Theorem C], [11, Theorem 5.3], as well as an L^2-energy gap result due to Nakajima [42, Corollary 1.2] for a Yang–Mills connection over the sphere, S^d, but with an arbitrary Riemannian metric.

☆ Paul Feehan was partially supported by National Science Foundation grant DMS-1510064, the Oswald Veblen Fund and Fund for Mathematics (Institute for Advanced Study, Princeton), and the Max Planck Institute for Mathematics, Bonn.

E-mail address: feehan@math.rutgers.edu.

http://dx.doi.org/10.1016/j.aim.2017.03.023

Contents

1. Introduction

1.1. Main result

The purpose of our article to establish the following

Theorem 1 *($L^{d/2}$-energy gap for Yang–Mills connections). Let G be a compact Lie group and P be a principal G-bundle over a closed, smooth manifold, X, of dimension $d \geq 2$ and endowed with a smooth Riemannian metric, g. Then there is a positive constant, $\varepsilon = \varepsilon(d, g, G) \in (0, 1]$, with the following significance. If A is a smooth Yang–Mills connection on P with respect to the metric, g, and its curvature, F_A, obeys*