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Energy gap for Yang–Mills connections, II: Arbitrary closed Riemannian manifolds[☆]



Paul M.N. Feehan

Department of Mathematics, Rutgers, The State University of New Jersey,
110 Frelinghuysen Road, Piscataway, NJ 08854-8019, United States

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ABSTRACT

We prove an $L^{d/2}$ energy gap result for Yang–Mills connections on principal G -bundles, P , over arbitrary, closed, Riemannian, smooth manifolds of dimension $d \geq 2$. We apply our version of the Łojasiewicz–Simon gradient inequality [16,19] to remove a positivity constraint on a combination of the Ricci and Riemannian curvatures in a previous $L^{d/2}$ -energy gap result due to Gerhardt [23, Theorem 1.2] and a previous L^∞ -energy gap result due to Bourguignon, Lawson, and Simons [10, Theorem C], [11, Theorem 5.3], as well as an L^2 -energy gap result due to Nakajima [42, Corollary 1.2] for a Yang–Mills connection over the sphere, S^d , but with an arbitrary Riemannian metric.

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E-mail address: feehan@math.rutgers.edu.

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1. Introduction

1.1. Main result

The purpose of our article to establish the following

Theorem 1 (*$L^{d/2}$ -energy gap for Yang–Mills connections*). *Let G be a compact Lie group and P be a principal G -bundle over a closed, smooth manifold, X , of dimension $d \geq 2$ and endowed with a smooth Riemannian metric, g . Then there is a positive constant, $\varepsilon = \varepsilon(d, g, G) \in (0, 1]$, with the following significance. If A is a smooth Yang–Mills connection on P with respect to the metric, g , and its curvature, F_A , obeys*

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