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Pointwise decay for the Maxwell field on black hole space–times [☆]

Jason Metcalfe [a], Daniel Tataru [b], Mihai Tohaneanu [c,*]

[a] *Department of Mathematics, University of North Carolina, Chapel Hill, NC 27599-3250, United States*
[b] *Department of Mathematics, University of California, Berkeley, CA 94720, United States*
[c] *Department of Mathematics, University of Kentucky, Lexington, KY 40506, United States*

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ABSTRACT

In this article we study the pointwise decay properties of solutions to the Maxwell system on a class of nonstationary asymptotically flat backgrounds in three space dimensions. Under the assumption that uniform energy bounds and a weak form of local energy decay hold forward in time, we establish peeling estimates, as well as a t^{-4} rate of decay on compact regions for all the components of the Maxwell tensor.

1. Introduction

In this article we consider the question of pointwise decay for solutions to the Maxwell system with localized initial data. The class of backgrounds we are interested in are

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[*] Corresponding author.

E-mail addresses: metcalfe@email.unc.edu (J. Metcalfe), tataru@math.berkeley.edu (D. Tataru), mihai.tohaneanu@uky.edu (M. Tohaneanu).

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certain asymptotically flat black hole backgrounds, e.g. of Schwarzschild/Kerr type and perturbations thereof. However, the type of results we obtain in this article treat a compact set essentially as a black box, so they also apply in other settings. Our interest in this problem originates from general relativity, where the Maxwell (or spin 1) system is a linearized model of the Einstein Equations that captures some of the difficulties not present in the scalar wave equation (or spin 0) case.

The main idea of this article is that the pointwise decay bounds are a consequence of local energy decay estimates for the same Maxwell system, even though the local energy decay bounds are invariant with respect to time translations, while the pointwise decay bounds are not. This fits into the philosophy that the local energy decay estimates are the core decay estimates, and the other types of decay estimates (e.g. Strichartz, pointwise) are derived bounds. In the context of the Schrödinger equation on asymptotically flat space–times, this approach was developed in [29,20]. More recently, the same philosophy was implemented in the context of the scalar wave equation, beginning with [22]. The case of the scalar wave equation on black hole space times is discussed in what follows.

We begin with local energy estimates for solutions to the scalar wave equation $\Box_g u = f$ on Schwarzschild and Kerr manifolds, which have been recently established by various authors ([4–6,8,9,21] for Schwarzschild, [30,10,1] for Kerr with small angular momentum, and [11–13] for Kerr with $|a| < M$). The transition from local energy decay to Strichartz estimates was considered in [21,31]. The key result that sharp decay bounds (Price's Law [25]) follow from the local energy decay was first obtained in [29] for stationary space–times, using time Fourier transform and resolvent analysis, and then in the nonstationary case in [23], by using more robust methods based on the classical vector field method. (See also [14,15] for a more refined Fourier based analysis applied to Schwarzschild space–times.)

The main result in the present article is the exact counterpart of [23] in the context of the Maxwell system, and asserts that local energy decay implies sharp[1] pointwise decay bounds. These can be seen as Price's law in the Maxwell setting; indeed, [26] conjectures a decay rate of t^{-5} in compact regions for the Maxwell system on the Schwarzschild metric.

Since our result is a conditional one, it is useful to review where we stand as far as local energy decay estimates are concerned. With regards to the Maxwell system on Schwarzschild, a class of local energy estimates (as well as some partial pointwise rates of decay) were established in [3] for solutions to the homogeneous system with no charge. For solutions to the homogeneous system on Kerr spacetimes with small angular momentum $|a| \ll M$ there is recent work [2] that establishes some local energy estimates and uniform energy bounds.

For the inhomogeneous system with charges, the article [28] provides local energy estimates in a variety of spherically symmetric spacetimes, including Schwarzschild. This

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[1] At least for $r \geq \frac{t}{2}$; understanding what happens in the interior of a small cone seems to be a more delicate matter.