

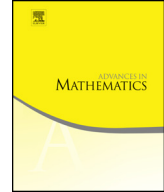


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



Non-compact Newton boundary and Whitney equisingularity for non-isolated singularities



Christophe Eyrals*, Mutsuo Okab

^a *Institute of Mathematics, Polish Academy of Sciences, Śniadeckich 8, 00-656 Warsaw, Poland*

^b *Department of Mathematics, Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan*

ARTICLE INFO

Article history:

Received 15 December 2015
Received in revised form 7 June 2017
Accepted 10 June 2017
Available online xxxx
Communicated by Karen Smith

MSC:

14J70
14J17
32S15
32S25

Keywords:

Non-isolated hypersurface singularities
Non-compact Newton boundary
Uniform local tameness
Whitney equisingularity

ABSTRACT

In an unpublished lecture note, J. Briançon observed that if $\{f_t\}$ is a family of isolated complex hypersurface singularities such that the Newton boundary of f_t is independent of t and f_t is non-degenerate, then the corresponding family of hypersurfaces $\{f_t^{-1}(0)\}$ is Whitney equisingular (and hence topologically equisingular). A first generalization of this assertion to families with non-isolated singularities was given by the second author under a rather technical condition. In the present paper, we give a new generalization under a simpler condition.

© 2017 Elsevier Inc. All rights reserved.

* Corresponding author.

E-mail addresses: eyralchr@yahoo.com (C. Eyrals), oka@rs.kagu.tus.ac.jp (M. Oka).

1. Introduction

Let $(t, \mathbf{z}) := (t, z_1, \dots, z_n)$ be coordinates for $\mathbb{C} \times \mathbb{C}^n$, let U be an open neighbourhood of $\mathbf{0} \in \mathbb{C}^n$ and D be an open disc centered at $0 \in \mathbb{C}$; finally, let

$$f: (D \times U, D \times \{\mathbf{0}\}) \rightarrow (\mathbb{C}, 0), (t, \mathbf{z}) \mapsto f(t, \mathbf{z}),$$

be a polynomial function. As usual, we write $f_t(\mathbf{z}) := f(t, \mathbf{z})$ and we denote by $V(f_t)$ the hypersurface in $U \subseteq \mathbb{C}^n$ defined by f_t . We are interested in the local structure of the singular loci of the hypersurfaces $V(f_t)$ at the origin $\mathbf{0} \in \mathbb{C}^n$ as the parameter t varies from a “small” non-zero value $t_0 \neq 0$ to $t = 0$. More precisely, we are looking for easy-to-check conditions on the members f_t of the family $\{f_t\}$ that guarantee equisingularity (in a sense to be specified) for the corresponding family of hypersurfaces $\{V(f_t)\}$.

In an unpublished lecture note [1], J. Briançon made the following observation.

Assertion 1.1 (Briançon). *Suppose that for all t sufficiently small, the following three conditions are satisfied:*

- (1) f_t has an isolated singularity at the origin $\mathbf{0} \in \mathbb{C}^n$;
- (2) the Newton boundary $\Gamma(f_t; \mathbf{z})$ of f_t at $\mathbf{0}$ with respect to the coordinates \mathbf{z} is independent of t ;
- (3) f_t is non-degenerate (in the sense of the Newton boundary as in [3,6]).

Then the family of hypersurfaces $\{V(f_t)\}$ is Whitney equisingular.

We say that a family $\{V(f_t)\}$ of (possibly non-isolated) hypersurface singularities is *Whitney equisingular* if there exists a Whitney stratification of the hypersurface $V(f) := f^{-1}(0)$ in an open neighbourhood \mathcal{U} of the origin $(0, \mathbf{0}) \in \mathbb{C} \times \mathbb{C}^n$ such that the t -axis $\mathcal{U} \cap (D \times \{\mathbf{0}\})$ is a stratum. By “Whitney stratification” we mean a Whitney stratification in the sense of [2]—that is, we do not require that the frontier condition holds. However, note that if \mathcal{S} is Whitney stratification of $\mathcal{U} \cap V(f)$ with the t -axis as a stratum, then so is the partition \mathcal{S}^c consisting of the connected components of the strata of \mathcal{S} ; moreover, \mathcal{S}^c satisfies the frontier condition (see [2] for details). Whitney equisingularity is quite a strong form of equisingularity. Combined with the Thom–Mather first isotopy theorem (cf. [2,4,12]), it implies topological equisingularity. Here, we say that the family $\{V(f_t)\}$ is *topologically equisingular* if for all sufficiently small t , there is an open neighbourhood $U_t \subseteq U$ of $\mathbf{0} \in \mathbb{C}^n$ together with a homeomorphism $\varphi_t: (U_t, \mathbf{0}) \rightarrow (\varphi_t(U_t), \mathbf{0})$ such that $\varphi_t(V(f_0) \cap U_t) = V(f_t) \cap \varphi_t(U_t)$. Note that a family of *isolated* hypersurface singularities (as in **Assertion 1.1**) is Whitney equisingular if and only if $V(f) \setminus (D \times \{\mathbf{0}\})$ is smooth and Whitney (b)-regular over $D \times \{\mathbf{0}\}$ in an open neighbourhood of the origin in $\mathbb{C} \times \mathbb{C}^n$. Here, it is worth to observe that, in general, even if the smooth part of $V(f)$ is Whitney (b)-regular along the t -axis, the family of

Download English Version:

<https://daneshyari.com/en/article/5778532>

Download Persian Version:

<https://daneshyari.com/article/5778532>

[Daneshyari.com](https://daneshyari.com)