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Global entropy solutions to weakly nonlinear gas dynamics

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ABSTRACT

Entropy weak solutions with bounded periodic initial data are considered for the system of weakly nonlinear gas dynamics. Through a modified Glimm scheme, an approximate solution sequence is constructed, and then a priori estimates are provided with the methods of approximate characteristics and approximate conservation laws, which gives not only the existence and uniqueness but also the uniform total variation bounds for the entropy solutions.

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1. Introduction

Consider the following system of weakly nonlinear gas dynamics.

$$\begin{cases} \partial_t\sigma_1 + \frac{\alpha}{2}\partial_x(\sigma_1^2) + \frac{\beta}{2}\int\limits_{-1}^{1}\frac{1}{2}\sigma_2'(\frac{x+y}{2})\sigma_3(y,t)\mathrm{d}y = 0, \\ \partial_t\sigma_3 - \frac{\alpha}{2}\partial_x(\sigma_3^2) - \frac{\beta}{2}\int\limits_{-1}^{1}\frac{1}{2}\sigma_2'(\frac{x+y}{2})\sigma_1(y,t)\mathrm{d}y = 0, \\ t=0:\ \sigma_1 = \sigma_{1,0}(x),\ \sigma_3 = \sigma_{3,0}(x), \end{cases} \tag{1.1}$$

where $\sigma_1 = \sigma_1(x,t)$ and $\sigma_3 = \sigma_3(x,t)$ are unknown functions, with $\sigma_{1,0}$ and $\sigma_{3,0}$ as given initial states satisfying

$$\sigma_{1,0}(x+1) = \sigma_{1,0}(x), \quad \sigma_{3,0}(x+1) = \sigma_{3,0}(x), \tag{1.2}$$

$$\int\limits_0^1 \sigma_{1,0}(x)\mathrm{d}x = 0, \quad \int\limits_0^1 \sigma_{3,0}(x)\mathrm{d}x = 0. \tag{1.3}$$

While, $\sigma_2 = \sigma_2(x)$ is any given $W^{1,1}_{\mathrm{loc}}$ function with

$$\sigma_2(x+\frac{1}{2}) = \sigma_2(x),^{3} \quad \int\limits_0^{\frac{1}{2}} |\sigma_2'(x)|\mathrm{d}x = M_E \leq \infty, \tag{1.4}$$

and α, β are given positive constants.

This system is first derived by A. Majda and R. Rosales in [19] from the 1-dimensional full Euler equations with *periodic* initial data through the method of weakly nonlinear geometric optics approximation to study the behavior of the solutions especially for the cases with resonance effects. Different from the Cauchy problem with initial data of small total variation, which has a quite complete theory for existence [12,24] and uniqueness [2], most aspects of the Cauchy problem for quasilinear systems of hyperbolic conservation laws with small periodic initial data are still open. One of the main difficulties may lie on the fact that the periodicity prevents the waves from separation and thus the system does not possess a decreasing Glimm functional to control the nonlinear effects, and almost all the classical analysis methods fail in this case. The celebrated work of J. Glimm–P.D. Lax [13] shows that for the isentropic Euler equations the Cauchy problem with small periodic initial data admits a global entropy solution. The method of their

[3] One may refer to [19] and [21], which show the reason why one needs only to count the even modes of σ_2.