



ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Double canonical bases [☆]Arkady Berenstein ^a, Jacob Greenstein ^{b,*}^a Department of Mathematics, University of Oregon, Eugene, OR 97403, USA^b Department of Mathematics, University of California, Riverside, CA 92521, USA

ARTICLE INFO

Article history:

Received 26 November 2015

Received in revised form 7 June 2017

Accepted 13 June 2017

Communicated by Roman

Bezrukavnikov

Keywords:

Quantum group

Canonical basis

Drinfeld double

Heisenberg double

Braided bialgebra

ABSTRACT

We introduce a new class of bases for quantized universal enveloping algebras $U_q(\mathfrak{g})$ and other doubles attached to semisimple and Kac–Moody Lie algebras. These bases contain dual canonical bases of upper and lower halves of $U_q(\mathfrak{g})$ and are invariant under many symmetries including all Lusztig’s symmetries if \mathfrak{g} is semisimple. It also turns out that a part of a double canonical basis of $U_q(\mathfrak{g})$ spans its center and consists of higher Casimirs which suggests physical applications.

© 2017 Elsevier Inc. All rights reserved.

Contents

1.	Introduction and main results	382
	Acknowledgments	391
2.	Equivariant Lusztig’s Lemma and bases of Heisenberg and Drinfeld doubles	391
2.1.	An equivariant Lusztig’s Lemma	391
2.2.	Double bases of Heisenberg and Drinfeld doubles	393
3.	Dual canonical bases and proofs of Theorems 1.3, 1.5, 1.10 and 1.19	399
3.1.	Bicharacters, pairings, lattices and inner products	399

[☆] This work was partially supported by the NSF grants DMS-1101507 and DMS-1403527 (A. B.) and by the Simons Foundation collaboration grant no. 245735 (J. G.).

* Corresponding author.

E-mail addresses: arkadiy@math.uoregon.edu (A. Berenstein), jacob.greenstein@ucr.edu (J. Greenstein).

<http://dx.doi.org/10.1016/j.aim.2017.06.005>

0001-8708/© 2017 Elsevier Inc. All rights reserved.

3.2.	Dual canonical bases	401
3.3.	Proofs of Theorems 1.3, 1.5 and 1.10	403
3.4.	Semi-classical limits	405
3.5.	Colored Heisenberg and quantum Weyl algebras and their bases	405
3.6.	Invariant quasi-derivations	407
4.	Examples of double canonical bases	411
4.1.	Double canonical basis of $U_q(\mathfrak{sl}_2)$	411
4.2.	Action on a double basis for \mathfrak{sl}_2	416
4.3.	Some elements in double canonical bases in ranks 2 and 3	418
4.4.	Reshetikhin–Semenov-Tian-Shansky map	424
4.5.	Towards Conjecture 1.26	429
5.	Bar-equivariant braid group actions	433
5.1.	Invariant braid group action on Drinfeld double	433
5.2.	Elements T_w , quantum Schubert cells and their bases	436
5.3.	Proof of Theorem 3.11	436
5.4.	Braid group action for $U_q(\mathfrak{sl}_2)$	437
5.5.	Braid group action on elements of \mathbf{B}_{n_+}	438
5.6.	Wild elements of a double canonical basis	442
Appendix A.	Drinfeld and Heisenberg doubles	442
A.1.	Nichols algebras	442
A.2.	Bar and star involutions	444
A.3.	Pairing and quasi-derivations	445
A.4.	Double smash products	448
A.5.	Bialgebra pairings and doubles of bialgebras	451
A.6.	Bosonization of Nichols algebras	452
A.7.	Drinfeld double	456
A.8.	Diagonal braidings	461
A.9.	Drinfeld double in the diagonal case	464
List of notation	467
References	467

1. Introduction and main results

The goal of this paper is to construct a canonical basis $\mathbf{B}_{\mathfrak{g}}$ of a quantized enveloping algebra $U_q(\mathfrak{g})$ where \mathfrak{g} is a semisimple or a Kac–Moody Lie algebra. For instance, if $\mathfrak{g} = \mathfrak{sl}_2$, then $\mathbf{B}_{\mathfrak{g}}$ is given by

$$\mathbf{B}_{\mathfrak{sl}_2} = \{q^{n(m_- - m_+)} K^n C^{(m_0)} F^{m_-} E^{m_+} \mid n \in \mathbb{Z}, m_0, m_{\pm} \in \mathbb{Z}_{\geq 0}, m_- \cdot m_+ = 0\}, \quad (1.1)$$

where we used a slightly non-standard presentation of $U_q(\mathfrak{sl}_2)$ obtained from the more familiar one by rescaling generators $E \mapsto (q^{-1} - q)E, F \mapsto (q - q^{-1})F$:

$$U_q(\mathfrak{sl}_2) := \langle E, F, K^{\pm 1} : KEK^{-1} = q^2E, KFK^{-1} = q^{-2}F, EF - FE = (q^{-1} - q)(K - K^{-1}) \rangle.$$

Here the $C^{(m)}$ are central elements of $U_q(\mathfrak{sl}_2)$ defined by $C^{(0)} = 1, C = C^{(1)} = EF - q^{-1}K - qK^{-1} = FE - qK - q^{-1}K^{-1}$ and $C \cdot C^{(m)} = C^{(m+1)} + C^{(m-1)}$ for $m \geq 1$.

We call $\mathbf{B}_{\mathfrak{sl}_2}$ double canonical because of the following remarkable properties (we will explain in §4.1 the reason why we must use Chebyshev polynomials $C^{(m)}$ instead of C^m).

Download English Version:

<https://daneshyari.com/en/article/5778539>

Download Persian Version:

<https://daneshyari.com/article/5778539>

[Daneshyari.com](https://daneshyari.com)