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Linear stability of compressible vortex sheets in two-dimensional elastodynamics

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A B S T R A C T

The linear stability of rectilinear compressible vortex sheets is studied for two-dimensional isentropic elastic flows. This problem has a free boundary and the boundary is characteristic. A necessary and sufficient condition is obtained for the linear stability of the rectilinear vortex sheets. More precisely, it is shown that, besides the stable supersonic zone, the elasticity exerts an additional stable subsonic zone. We also find that there is a class of states in the interior of subsonic zone where the stability of such states is weaker than the stability of other states in the sense that there is an extra loss of tangential derivatives with respect to the source terms. This is a new feature that the Euler flow does not possess. One of the difficulties for the elastic flow is that the non-differentiable points of the eigenvalues may coincide with the roots of the Lopatinskii determinant. As a result, the Kreiss symmetrization cannot be applied directly. Instead, we perform an upper triangularization of the system to separate only the outgoing modes at all points in the frequency space, so that an exact estimate of the outgoing modes can be obtained. Moreover, all the outgoing modes are shown to be zero due to the L^2-regularity of solutions. The estimates for the incoming modes can be derived directly from the Lopatinskii determinant. This new approach avoids the lengthy computation and estimates for the outgoing modes

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when Kreiss symmetrization is applied. This method can also be applied to the Euler flow and MHD flow.

Contents

1. Introduction

Vortex sheets are interfaces between two incompressible or compressible flows passing along each other. They arise in a broad range of physical problems in fluid mechanics, aerodynamics, oceanography and astrophysical plasma. Some typical examples include the sharp interface between two parallel shear flows, and vortex flows where the vortices are concentrated within a thin layer. In particular, for compressible flows, vortex sheets are fundamental waves which play an important role in the study of general entropy solutions to multi-dimensional hyperbolic systems of conservation laws. Analyzing the existence and stability of compressible vortex sheets may shed light on the understanding of fluid dynamics and the behavior of entropy solutions.

In this paper, we are concerned with the vortex sheets in the following two-dimensional compressible inviscid flow in elastodynamics [18,22,31]:

$$\rho_t + \mathrm{div}(\rho \mathbf{u}) = 0, \tag{1.1}$$

$$(\rho \mathbf{u})_t + \mathrm{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p = \mathrm{div}(\rho \mathtt{F}\mathtt{F}^\top), \tag{1.2}$$

$$\mathtt{F}_t + \mathbf{u} \cdot \nabla \mathtt{F} = \nabla \mathbf{u} \mathtt{F}, \tag{1.3}$$