

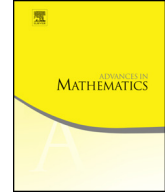


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Higher regularity of the free boundary in the obstacle problem for the fractional Laplacian



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ABSTRACT

We prove a higher regularity result for the free boundary in the obstacle problem for the fractional Laplacian via a higher order boundary Harnack estimate.

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1. Introduction and main results

In this paper, we investigate the higher regularity of the free boundary in the fractional obstacle problem. We prove a higher order boundary Harnack estimate, building on ideas developed by De Silva and Savin in [12–14]. As a consequence, we show that if the obstacle is $C^{m,\beta}$, then the free boundary is $C^{m-1,\alpha}$ near regular points for some $0 < \alpha \leq \beta$. In particular, smooth obstacles yield smooth free boundaries near regular points.

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1.1. *The fractional obstacle problem*

For a given function (obstacle) $\varphi \in C(\mathbb{R}^n)$ decaying rapidly at infinity and $s \in (0, 1)$, a function v is a solution of the fractional obstacle problem if

$$\begin{cases} v(x) \geq \varphi(x) & \text{in } \mathbb{R}^n \\ \lim_{|x| \rightarrow \infty} v(x) = 0 & \text{on } \mathbb{R}^n \\ (-\Delta)^s v(x) \geq 0 & \text{in } \mathbb{R}^n \\ (-\Delta)^s v(x) = 0 & \text{in } \{v > \varphi\} \end{cases} \tag{1.1}$$

where the s -Laplacian $(-\Delta)^s$ of a function u is defined by

$$(-\Delta)^s u(x) := c_{n,s} \text{PV} \int_{\mathbb{R}^n} \frac{u(x) - u(x+z)}{|z|^{n+2s}} dz.$$

The sets

$$\mathcal{P} := \{v = \varphi\} \quad \text{and} \quad \Gamma := \partial\{v = \varphi\}$$

are known as the *contact set* and the *free boundary* respectively.

The fractional obstacle problem appears in many contexts, including the pricing of American options with jump processes (see [11] and the Appendix of [5] for an informal discussion) and the study of the regularity of minimizers of nonlocal interaction energies in kinetic equations (see [10]).

While the obstacle problem for the fractional Laplacian is nonlocal, it admits a local formulation thanks to the extension method (see [9,24]). Specifically, one considers the a -harmonic¹ extension \tilde{v} of v to the upper half-space $\mathbb{R}_+^{n+1} := \mathbb{R}^n \times (0, \infty)$:

$$\begin{cases} L_a \tilde{v}(x, y) = 0 & \text{in } \mathbb{R}_+^{n+1} \\ \tilde{v}(x, 0) = v(x) & \text{on } \mathbb{R}^n \end{cases}$$

where

$$L_a u(x, y) := \text{div}(|y|^a \nabla u(x, y)) \quad \text{and} \quad a := 1 - 2s \in (-1, 1).$$

The function \tilde{v} is obtained as the minimizer of the variational problem

$$\min \left\{ \int_{\mathbb{R}_+^{n+1}} |\nabla u|^2 |y|^a dx dy : u \in H^1(\mathbb{R}_+^{n+1}, |y|^a), u(x, 0) = v(x) \right\}$$

¹ We say a function u is a -harmonic if $L_a u = 0$.

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