Advances in Mathematics 311 (2017) 748–795

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Higher regularity of the free boundary in the obstacle problem for the fractional Laplacian

Yash Jhaveri, Robin Neumayer *

The University of Texas at Austin, Department of Mathematics, Austin, TX 78712, USA

ARTICLE INFO

Article history:
Received 15 June 2016
Received in revised form 23 February 2017
Accepted 6 March 2017
Communicated by Ovidiu Savin

Keywords:
Boundary Harnack estimate
Fractional Laplacian
Free boundary problems

ABSTRACT

We prove a higher regularity result for the free boundary in the obstacle problem for the fractional Laplacian via a higher order boundary Harnack estimate.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction and main results

In this paper, we investigate the higher regularity of the free boundary in the fractional obstacle problem. We prove a higher order boundary Harnack estimate, building on ideas developed by De Silva and Savin in [12–14]. As a consequence, we show that if the obstacle is $C^{m,\beta}$, then the free boundary is $C^{m-1,\alpha}$ near regular points for some $0 < \alpha \leq \beta$. In particular, smooth obstacles yield smooth free boundaries near regular points.

* Corresponding author.
E-mail addresses: yjhaveri@math.utexas.edu (Y. Jhaveri), rneumayer@math.utexas.edu (R. Neumayer).

http://dx.doi.org/10.1016/j.aim.2017.03.006
0001-8708/© 2017 Elsevier Inc. All rights reserved.

1.1. The fractional obstacle problem

For a given function (obstacle) $\varphi \in C(\mathbb{R}^n)$ decaying rapidly at infinity and $s \in (0, 1)$, a function v is a solution of the fractional obstacle problem if

$$\begin{cases} v(x) \geq \varphi(x) & \text{in } \mathbb{R}^n \\ \lim_{|x|\to\infty} v(x) = 0 & \text{on } \mathbb{R}^n \\ (-\Delta)^s v(x) \geq 0 & \text{in } \mathbb{R}^n \\ (-\Delta)^s v(x) = 0 & \text{in } \{v > \varphi\} \end{cases} \tag{1.1}$$

where the s-Laplacian $(-\Delta)^s$ of a function u is defined by

$$(-\Delta)^s u(x) := c_{n,s} \, \mathrm{PV} \int_{\mathbb{R}^n} \frac{u(x) - u(x+z)}{|z|^{n+2s}} \, \mathrm{d}z.$$

The sets

$$\mathcal{P} := \{v = \varphi\} \qquad \text{and} \qquad \Gamma := \partial\{v = \varphi\}$$

are known as the *contact set* and the *free boundary* respectively.

The fractional obstacle problem appears in many contexts, including the pricing of American options with jump processes (see [11] and the Appendix of [5] for an informal discussion) and the study of the regularity of minimizers of nonlocal interaction energies in kinetic equations (see [10]).

While the obstacle problem for the fractional Laplacian is nonlocal, it admits a local formulation thanks to the extension method (see [9,24]). Specifically, one considers the a-harmonic[1] extension $\tilde{v}$ of v to the upper half-space $\mathbb{R}^{n+1}_+ := \mathbb{R}^n \times (0, \infty)$:

$$\begin{cases} L_a \tilde{v}(x, y) = 0 & \text{in } \mathbb{R}^{n+1}_+ \\ \tilde{v}(x, 0) = v(x) & \text{on } \mathbb{R}^n \end{cases}$$

where

$$L_a u(x, y) := \mathrm{div}(|y|^a \nabla u(x, y)) \qquad \text{and} \qquad a := 1 - 2s \in (-1, 1).$$

The function $\tilde{v}$ is obtained as the minimizer of the variational problem

$$\min \left\{ \int_{\mathbb{R}^{n+1}_+} |\nabla u|^2 \, |y|^a \mathrm{d}x \, \mathrm{d}y \, : \, u \in H^1(\mathbb{R}^{n+1}_+, |y|^a), \, u(x, 0) = v(x) \right\}$$

[1] We say a function u is *a-harmonic* if $L_a u = 0$.

Download English Version:

https://daneshyari.com/en/article/5778576

Download Persian Version:

https://daneshyari.com/article/5778576

Daneshyari.com