

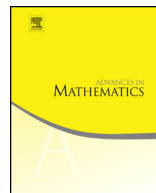


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Surfaces of minimal degree of tame representation type and mutations of Cohen–Macaulay modules [☆]



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ABSTRACT

We provide two examples of smooth projective surfaces of tame CM type, by showing that the parameter space of isomorphism classes of indecomposable ACM bundles with fixed rank and determinant on a rational quartic scroll in $\mathbb{P}^5$ is either a single point or a projective line. These turn out to be the only smooth projective ACM varieties of tame CM type besides elliptic curves, [1].

For surfaces of minimal degree and wild CM type, we classify rigid Ulrich bundles as Fibonacci extensions. For $\mathbb{F}_0$ and $\mathbb{F}_1$, embedded as quintic or sextic scrolls, a complete classification of rigid ACM bundles is given.

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Introduction

Let $X \subset \mathbb{P}^n$ be a smooth positive-dimensional closed subvariety over an algebraically closed field $\mathbb{k}$, and assume that the graded coordinate ring $\mathbb{k}[X]$ of X is Cohen–Macaulay, i.e. X is ACM (arithmetically Cohen–Macaulay). Then X supports infinitely many indecomposable ACM sheaves $\mathcal{E}$ (i.e. whose $\mathbb{k}[X]$ -module of global sections $H_*^0(X, \mathcal{E})$ is a maximal Cohen–Macaulay), unless X is $\mathbb{P}^n$ itself, or a quadric hypersurface, or a rational normal curve, or one of the two sporadic cases: the Veronese surface in $\mathbb{P}^5$ and (cf. §1.2) the rational cubic scroll $S(1, 2)$ in $\mathbb{P}^4$, see [12].

Actually, for most ACM varieties X , much more is true. Namely X supports families of arbitrarily large dimension of indecomposable ACM bundles, all non-isomorphic to one another (varieties like this are of “geometrically of wild CM type” or simply “CM-wild”). CM-wild varieties include curves of genus ≥ 2 , hypersurfaces of degree $d \geq 4$ in $\mathbb{P}^n$ with $n \geq 2$, complete intersections in $\mathbb{P}^n$ of codimension ≥ 3 , having one defining polynomial of degree ≥ 3 (cf. [9,10]), the third Veronese embedding of any variety of dimension ≥ 2 cf. [24]. In many cases, these families are provided by Ulrich bundles, i.e. those $\mathcal{E}$ such that $H_*^0(X, \mathcal{E})$ achieves the maximum number of generators, namely $d_X \operatorname{rk}(\mathcal{E})$, where we write d_X for the degree of X . For instance, Segre embeddings are treated in [8], smooth rational ACM surfaces in $\mathbb{P}^4$ in [25], cubic surfaces and threefolds in [5,6], del Pezzo surfaces in [27,7].

In spite of this, there is a special class of varieties X with intermediate behaviour, namely X supports continuous families of indecomposable ACM bundles, all non-isomorphic to one another, but, for each rank r , these bundles form finitely (or countably) many irreducible families of dimension at most one. Then X is called of tame CM type, or of tame representation type. It is the case of the elliptic curve, [1].

In this note we provide the first examples of smooth positive-dimensional projective CM-tame varieties, besides elliptic curves. Part of this was announced in [15].

Theorem A. *Let X be a smooth surface of degree 4 in $\mathbb{P}^5$. Then, for any $r \geq 1$, there is a family of isomorphism classes of indecomposable Ulrich bundles of rank $2r$, parametrized by $\mathbb{P}^1$. Conversely, any indecomposable ACM bundle on X is rigid or belongs to one of these families (up to a twist). In particular, X is of tame CM type.*

Recalling the classification by del Pezzo and Bertini of smooth varieties of *minimal degree*, i.e. with $d_X = \operatorname{codim}(X) + 1$, as the Veronese surface in $\mathbb{P}^5$ and rational normal scrolls (cf. [11]), we see two things. On one hand, both CM-finite varieties and our examples have minimal degree, actually a surface of degree 4 in $\mathbb{P}^5$ is a quartic scroll. Incidentally, these have the same graded Betti numbers as the Veronese surface in $\mathbb{P}^5$, which is CM-finite. On the other hand the remaining varieties of minimal degree are CM-wild by [23]; in fact in [23] also quartic scrolls are claimed to be of wild CM type: the gap in the argument only overlooks our two examples, cf. Remark 2.2. By the following result, these examples complete the list of non-CM-wild varieties in a broad sense.

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