

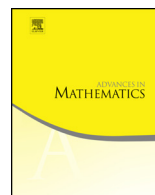


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# The large scale geometry of strongly aperiodic subshifts of finite type



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## ABSTRACT

A subshift on a group  $G$  is a closed,  $G$ -invariant subset of  $A^G$ , for some finite set  $A$ . It is said to be a subshift of finite type (SFT) if it is defined by a finite collection of “forbidden patterns”, to be strongly aperiodic if all point stabilizers are trivial, and weakly aperiodic if all point stabilizers are infinite index in  $G$ . We show that groups with at least 2 ends have a strongly aperiodic SFT, and that having such an SFT is a QI invariant for finitely presented groups. We show that a finitely presented torsion free group with no weakly aperiodic SFT must be QI-rigid. The domino problem on  $G$  asks whether the SFT specified by a given set of forbidden patterns is empty. We show that decidability of the domino problem is a QI invariant.

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## 1. Introduction

Recall that a topological dynamical system is a pair  $(\Omega, G)$  where  $G$  is a group acting by homeomorphisms on the compact space  $\Omega$ . For instance, if  $A$  is a finite discrete set, then the group  $G$  acts (on the right) on the compact space  $A^G$  by homeomorphisms via

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$$(\sigma \cdot h)(g) = \sigma(hg).$$

This action makes the pair  $(A^G, G)$  into a topological dynamical system called the right shift. When  $G = \mathbb{Z}$ , an element  $h$  of  $G$  acts on a biinfinite word  $\sigma \in A^G$  by “shifting” it, whence the name. A closed,  $G$ -invariant subset of  $A^G$  is known as a subshift (for reference, see [7] or [8]). To say that a subshift  $X$  codes a dynamical system  $(\Omega, G)$  means that there exists a continuous  $G$ -equivariant surjection from  $X$  to  $\Omega$ .

**Subshifts of finite type (see [8, §2]).** How would one construct a subshift? The simplest idea is to start with a closed set  $C$  of  $A^G$  and intersect its  $G$ -translates. The most important case of this construction arises when  $C$  is determined by finitely many coordinates.

**Definition 1.1.** Let  $A$  be a finite set and  $G$  a group. If  $S$  is a finite subset of  $G$  and  $L$  a subset of  $A^S$ , then the clopen set

$$\{\sigma \in A^G : \sigma|_S \notin L\}$$

is known as a cylinder set. If  $C$  is a cylinder set, then the set  $X$  given by  $\bigcap_{g \in G} (C \cdot g)$  is called a subshift of finite type. We say that  $X$  is defined on  $S$ . If  $F$  is a finite set, then  $\alpha \in A^F$  is called a forbidden pattern for  $X$  if it is never equal to  $(\sigma \cdot g)|_F$  for any  $\sigma \in X$ .

We say that a subshift of finite type  $X \subseteq A^G$  is defined by a finite collection  $\mathcal{F}$  of forbidden patterns  $\alpha_i : F_i \rightarrow A$  if  $X$  is exactly the set of  $\sigma \in A^G$  such that  $(\sigma \cdot g)|_{F_i}$  is not equal to  $\alpha_i$  for any  $i$  and any  $g \in G$ —i.e., if

$$X = \bigcap_{g \in G} \bigcap_{\alpha \in \mathcal{F}} \{\sigma \in A^G : \sigma|_{gF} \neq \alpha\} \cdot g.$$

*1.1. Aperiodicity and the domino problem*

Given a finite set of forbidden patterns  $\mathcal{F}$ , it is entirely possible that the subshift of finite type defined by  $\mathcal{F}$  is empty.

**Definition 1.2.** Let  $G$  be a finitely generated group. We say that  $G$  has decidable domino problem if there exists an algorithm which takes as input a finite set of forbidden patterns  $\mathcal{F}$  and determines whether the subshift they define is empty.

**The domino problem for  $\mathbb{Z}$ .** Suppose we are given a finite set of forbidden patterns  $\mathcal{F}$  defining a subshift of finite type  $X_{\mathcal{F}}$  over  $\mathbb{Z}$ . By compactness  $X_{\mathcal{F}}$  is empty only if there is some  $n$  such that every  $A$ -coloring of the  $n$ -ball in  $\mathbb{Z}$  includes some forbidden pattern. Hence, if  $X_{\mathcal{F}}$  is empty, a Turing machine may discover this fact in finite time.

On the other hand, if  $X_{\mathcal{F}}$  is nonempty, one would like to certify nonemptiness by finding a “constructible”  $\sigma \in A^G$  which can be proven to lie in  $X_{\mathcal{F}}$ . The simplest constructible elements of  $A^{\mathbb{Z}}$  are periodic—i.e., fixed by translation by some  $n$  and hence of

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