

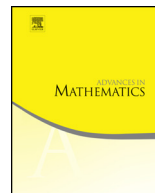


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A spectral sequence for polyhedral products

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This paper is dedicated to our friend and colleague Samuel Gitler

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ABSTRACT

The purpose of this paper is to exhibit fine structure for polyhedral products $Z(K; (\underline{X}, \underline{A}))$ and polyhedral smash products $\widehat{Z}(K; (\underline{X}, \underline{A}))$ where moment-angle complexes are special cases in which $(X, A) = (D^2, S^1)$. There are three main parts to this paper.

- (1) One part gives a natural filtration of the polyhedral product together with properties of the resulting spectral sequence in [Theorem 2.15](#). Applications of this spectral sequence are given.
- (2) The second part gives a homological decomposition of $\widehat{Z}(K; (\underline{X}, \underline{A}))$ in case $(\underline{X}, \underline{A})$ consists of CW pairs by using part 1.
- (3) Applications to the ring structure of $Z(K; (\underline{X}, \underline{A}))$ are given for CW-pairs (X, A) satisfying suitable freeness conditions.

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1. Introduction

The subject of this paper is the homology of polyhedral products $Z(K; (\underline{X}, \underline{A}))$, and polyhedral smash products $\widehat{Z}(K; (\underline{X}, \underline{A}))$ [9,5,1–3]. Definitions are listed in section 2 of this paper.

One of the purposes of this article is to give the Hilbert–Poincaré series for the polyhedral product $Z(K; (\underline{X}, \underline{A}))$ in terms of

- (1) the kernel, image, and cokernel of the induced maps

$$H^*(X_i) \rightarrow H^*(A_i)$$

for all i , and

- (2) the full sub-complexes of K .

This computation was also worked out in [4]¹ using more geometric methods.

This is achieved by analysis of a spectral sequence abutting to the cohomology of the polyhedral product $Z(K; (\underline{X}, \underline{A}))$ by filtering this space with the left-lexicographical ordering of simplices. The method applies to a generalized multiplicative cohomology theory, h^* as well. The spectral sequence is then used to describe some features of the ring structure of $h^*(Z(K; (\underline{X}, \underline{A})))$.

Qibing Zheng [12] gives an alternative description of the cohomology of a polyhedral product. Our methods are distinct from his and the presentation of the computational results assumes a different form. Unlike the spectral sequence developed here, his collapses at the E_2 term.

2. Definitions, and main results

The basic constructions addressed in this article are defined in this section. First recall the definition of an abstract simplicial complex.

Definition 2.1.

- (1) Let K denote an abstract simplicial complex with m vertices labeled by the set $[m] = \{1, 2, \dots, m\}$. Thus, K is a subset of the power set of $[m]$ such that an element given by a $(k - 1)$ -simplex σ of K is given by an ordered sequence $\sigma = (i_1, \dots, i_k)$ with $1 \leq i_1 < \dots < i_k \leq m$ such that if $\tau \subset \sigma$, then τ is a simplex of K . In particular the empty set \emptyset is a subset of σ and so it is in K . The vertex set of σ $\{i_1, \dots, i_k\}$ will be denoted $v(\sigma)$.

¹ While the third and fourth author were visiting the Institute for Advanced Study.

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