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A well-posedness theory for the Prandtl equations in three space variables

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ABSTRACT

The well-posedness of the three space dimensional Prandtl equations is studied under some constraint on its flow structure. Together with the instability result given in [28], it gives an almost necessary and sufficient structural condition for the stability of the three-dimensional Prandtl equations. It reveals that the classical Burgers equation plays an important role in determining this type of flow with special structure, that avoids the appearance of the secondary flow, an unstabilizing factor in the three-dimensional Prandtl boundary layers. And the sufficiency of the monotonicity condition on the tangential velocity field for the existence of solutions to the Prandtl boundary layer equations is illustrated in the three-dimensional setting. Moreover, it is shown that this structured flow is linearly stable for any smooth three-dimensional perturbation.

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Contents

1. Introduction

To describe the behavior of viscous flows in a neighborhood of physical boundary qualitatively and quantitatively is a classical problem both in theoretical and applied fluid mechanics. It was observed by L. Prandtl in his seminal work [36] that, away from the boundary the flow is mainly driven by convection so that the viscosity can be negligible, while in a small neighborhood of physical boundary the effect of the viscosity plays a significant role in the flow. Hence, there exists a thin transition layer near the boundary, in which the behavior of flow changes dramatically, this transition layer is so-called the boundary layer.

Mathematically, taking the incompressible Navier–Stokes equations as the governed system for the viscous flow with velocity being non-slip on the boundary, in Prandtl's theory, letting ν be the kinematic viscosity, outside the layer of thickness $\sqrt{\nu}$ near the boundary, the flow is approximated by an inviscid one, and it is basically governed by the incompressible Euler equations; on the other hand, inside the layer, the convection and the viscosity balance so that the flow can be modeled by a system derived from the Navier–Stokes equations by asymptotic expansion, that is, the Prandtl boundary layer equations. The formal derivation of the Prandtl equations can be found in [36], for example.

In the Prandtl boundary layer equations, the tangential velocity profile satisfies a system of nonlinear degenerate parabolic equations, and the incompressibility of flow still holds in the layer, so the tangential and normal velocities are coupled by the divergence-free constraint. The main difficulties in studying the Prandtl equations lie in the degeneracy, mixed type, nonlinearity and non-local effect in the system, so that the classical mathematical theories of partial differential equations can hardly be applied. For this, in more than one hundred years since the Prandtl equations were derived, there is still no general mathematical theory on its well-posedness except in the framework of analytic functions by using the abstract Cauchy–Kowaleskaya theory [37]. In recent years, by using the analytic approach [37] or the energy estimates, there are further works concerned with the well-posedness of the Prandtl equations in the space of analytic functions (cf. [6,23,26,29,43] etc.), and in the Gevrey class [15]. However, the

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