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Landau–Ginzburg/Calabi–Yau correspondence for the complete intersections $X_{3,3}$ and $X_{2,2,2,2}$

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ABSTRACT

We define a generalization of Fan–Jarvis–Ruan–Witten theory, a “hybrid” model associated to a collection of quasihomogeneous polynomials of the same weights and degree, which is expected to match the Gromov–Witten theory of the Calabi–Yau complete intersection cut out by the polynomials. In genus zero, we prove that the correspondence holds for any such complete intersection of dimension three in ordinary, rather than weighted, projective space. These results generalize those of Chiodo–Ruan for the quintic threefold, and as in that setting, Givental’s quantization can be used to yield a conjectural relation between the full higher-genus theories.

1. Introduction

In the early 1990s, when the mathematical study of mirror symmetry was just beginning, physicists posited the existence of a Landau–Ginzburg/Calabi–Yau (LG/CY) correspondence connecting the geometry of Calabi–Yau complete intersections in projective space to the Landau–Ginzburg model, in which the polynomials defining the

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complete intersections are studied as singularities instead [19,20]. Mathematically, the theory on the CY side is the Gromov–Witten theory of the complete intersection, but it was not until 2007 with the series of papers [11–13] that a candidate theory on the LG side was suggested, namely Fan–Jarvis–Ruan–Witten (FJRW) theory. In [7], the Gromov–Witten theory of the quintic threefold was shown to match the FJRW theory of the corresponding singularity in genus zero.

The goal of this paper is to extend the results of [7] to certain complete intersections in projective space. In order to accomplish this, it is necessary to generalize FJRW theory, constructing a mathematical Landau–Ginzburg model associated to a collection of singularities rather than just one. The theory we construct is a "hybrid" model that combines aspects of FJRW theory and Gromov–Witten theory.

The idea for the hybrid model, as well as the technical tools required for its development, were already known by a number of authors, and this paper owes a great debt to them. The initial definition of the hybrid moduli space was suggested by A. Chiodo, who explained it to the author and proposed the project on which this work is based. To define a virtual cycle for the theory, we use the method of cosection localization, which is due to Kiem–Li–Chang [2,16]. Our application of the cosection method closely follows the work of Chang–Li [2]; in fact, for the case of the quintic threefold, the construction considered in this paper is nothing but the Landau–Ginzburg analogue of their argument. The first application of cosection localization to the Landau–Ginzburg side is due to Chang–Li–Li, who used it in [3] to give an algebraic construction of FJRW theory in the case of narrow sectors. This paper can be considered a generalization of their results.

Via Givental's quantization machinery, the genus-zero LG/CY correspondence yields a conjectural relationship between the hybrid model and the Gromov–Witten theory of the complete intersection in higher genus. While computations of Gromov–Witten theory past genus 1 are currently beyond the scope of mathematicians' methods, the Landau–Ginzburg model is generally thought to be more computationally manageable [7]. Thus, if the higher-genus correspondence could be verified, it would potentially open exciting avenues for Gromov–Witten theory.

1.1. Main result

Associated to a nondegenerate collection of quasihomogeneous polynomials $W_1, \ldots, W_r \in \mathbb{C}[x_1, \ldots, x_N]$, each with weights $c_1, \ldots, c_N$ and degree d satisfying the Calabi–Yau condition

$$dr = \sum_{j=1}^{N} c_j, \tag{1}$$

there are two associated theories. On the Calabi–Yau side, one considers the complete intersection X in weighted projective space cut out by the polynomials. The cohomology of this complete intersection can be viewed as the state space from which insertions