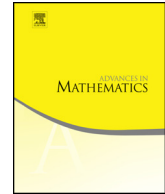




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Subdyadic square functions and applications to weighted harmonic analysis [☆]

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ABSTRACT

Through the study of novel variants of the classical Littlewood–Paley–Stein g -functions, we obtain pointwise estimates for broad classes of highly-singular Fourier multipliers on \mathbb{R}^d satisfying regularity hypotheses adapted to fine (subdyadic) scales. In particular, this allows us to efficiently bound such multipliers by geometrically-defined maximal operators via general weighted L^2 inequalities, in the spirit of a well-known conjecture of Stein. Our framework applies to solution operators for dispersive PDE, such as the time-dependent free Schrödinger equation, and other highly oscillatory convolution operators that fall well beyond the scope of the Calderón–Zygmund theory.

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1. Introduction

A common feature of many themes in both classical and contemporary harmonic analysis is the pivotal role played by operators which exhibit a certain *quadratic* structure. Such operators are usually referred to as *square functions*, and their study has its roots

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in the classical Littlewood–Paley theory (see for example [36,37,39]). A common role of the square functions is to capture manifestations of orthogonality in L^p spaces for $p \neq 2$. A primordial example is the square function (or *g-function*)

$$g(f)(x) := \left(\int_0^\infty \left| \frac{\partial u}{\partial t}(x, t) \right|^2 t dt \right)^{1/2}, \quad (1)$$

where $u : \mathbb{R}^d \times \mathbb{R}_+ \rightarrow \mathbb{R}$ denotes the Poisson integral of the function f on \mathbb{R}^d . While Plancherel’s theorem quickly reveals that $\|g(f)\|_2 \equiv \|f\|_2$, the key point is that this property essentially persists on L^p – that is, the norms $\|g(f)\|_p$ and $\|f\|_p$ are equivalent for all $1 < p < \infty$. Such facts have many important consequences, making square functions a central tool in modern analysis and PDE. In particular, square functions play a striking role in the classical theory of Fourier multipliers. On an abstract level, this approach to multipliers, originating in fundamental work of Stein [36], consists of identifying square functions g_1 and g_2 for which we have the *pointwise* estimate

$$g_1(T_m f)(x) \lesssim g_2(f)(x); \quad (2)$$

here T_m denotes the convolution operator with Fourier multiplier m .¹ Given such an estimate one may then deduce bounds on T_m from bounds on the square functions g_1 and g_2 . More specifically, if one has

$$\|f\|_X \lesssim \|g_1(f)\|_Y \quad \text{and} \quad \|g_2(f)\|_Y \lesssim \|f\|_Z, \quad (3)$$

for suitable normed spaces X, Y, Z , then the pointwise estimate (2) quickly reveals that

$$\|T_m f\|_X \lesssim \|g_1(T_m f)\|_Y \lesssim \|g_2(f)\|_Y \lesssim \|f\|_Z; \quad (4)$$

that is, T_m is bounded from Z to X .² The prime example of this approach in action is Stein’s celebrated proof of the classical Hörmander–Mikhlin multiplier theorem, which states that if a Fourier multiplier m on \mathbb{R}^d satisfies

$$\sup_{r>0} \|m(r \cdot) \Psi\|_{H^\sigma} < \infty \quad (5)$$

for some $\sigma > d/2$, or equivalently

$$\sup_{r>0} r^\theta r^{-d/2} \|m \Psi(r^{-1} \cdot)\|_{\dot{H}^\theta} < \infty \quad (6)$$

¹ Throughout this paper we shall write $A \lesssim B$ if there exists a constant c such that $A \leq cB$. In particular, this constant will always be independent of the input function f , variable x and weight function w . The relations $A \gtrsim B$ and $A \sim B$ are defined similarly.

² Of course this requires that the norm $\|\cdot\|_Y$ is increasing in the sense that $f_1 \lesssim f_2 \implies \|f_1\|_Y \lesssim \|f_2\|_Y$.

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