

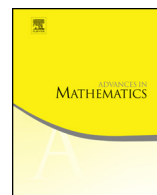


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Stable basic sets for finite special linear and unitary groups



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ABSTRACT

In this paper we show, using Deligne–Lusztig theory and Kawanaka’s theory of generalised Gelfand–Graev representations, that the decomposition matrix of the special linear and unitary group in non-defining characteristic can be made unitriangular with respect to a basic set that is stable under the action of automorphisms.

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1. Introduction

While ordinary characters of finite groups of Lie type are fairly well known, not so much is known about Brauer characters and in particular we do not have a general parameterisation for them. Decomposition matrices offer information about Brauer characters, and the unitriangularity of such matrices sets up a natural bijection from the set of Brauer characters to a corresponding so-called basic set of ordinary characters (see [Definition 2.2](#)). When we have such a bijection, we can try to obtain an equivariant one with respect to automorphisms. This is the case if and only if the basic set is stable

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under the action of automorphisms (see [Lemma 2.3](#)). This is useful to deal with some counting conjectures (see [\[4, Theorem 7.4\]](#)).

The unitriangularity of the decomposition matrices of finite general linear groups $GL_n(q)$ in non-defining characteristic ℓ was proved by Dipper in [\[6\]](#) and [\[7\]](#), while the case of special linear groups $SL_n(q)$ was done by Kleshchev and Tiep in [\[14\]](#). However the techniques used in these papers rely on an actual construction of Brauer characters of $GL_n(q)$. Such a construction is not known in the case of general unitary groups $GU_n(q)$ so these methods cannot be applied for now to prove the unitriangularity result for either $GU_n(q)$ or the special unitary groups $SU_n(q)$. Nevertheless the result has been shown for $GU_n(q)$ by Geck in [\[9\]](#), using Kawanaka’s theory of Generalised Gelfand Graev Representations (GGGRs for short). Such a method can also be used to recover the result for $GL_n(q)$. In [\[10\]](#) the same method was applied in the case of $SU_n(q)$, but only the cases where $\ell \nmid \gcd(n, q+1)$ could be treated (see [\[10, Theorem C\]](#)). In this case the usual basic set for $GU_n(q)$ gives by restriction a basic set for $SU_n(q)$. Investigating the methods developed in [\[14\]](#) and translating them in the context of Deligne–Lusztig theory and GGGRs, we first found that their methods could be adapted to $SU_n(q)$. But as was the case for $SL_n(q)$, the many basic sets obtained for $SU_n(q)$ were not stable with respect to automorphisms. In this paper we prove the following stronger statement:

Theorem A. *Let q be a power of some prime number p and ℓ be a prime number not dividing q . Let $n \geq 2$ and let $G \in \{SL_n(q), SU_n(q)\}$ be either the special linear or unitary group over a finite field with q elements. Then G has a unitriangular basic set in characteristic ℓ that is stable under the action of $\text{Aut}(G)$.*

Let $\tilde{G} = GL_n(q)$ (resp. $GU_n(q)$) and $G = SL_n(q)$ (resp. $G = SU_n(q)$). The unitriangular basic set that we obtain for G is explicitly built from a unitriangular basic set for \tilde{G} . We develop in the first part, in a general setting, a condition that ensures the existence of a stable unitriangular basic set for G provided that we already have one for \tilde{G} . The second part recalls some known facts about the character theory of general linear and unitary groups, and the last part is devoted to proving [Theorem A](#).

2. Stable unitriangular basic set for a normal subgroup

2.1. Decomposition numbers, unitriangularity and stability under group action

Our basic reference for the representation theory of finite groups is [\[20\]](#). Let H be a finite group and ℓ be a prime number dividing the order of H . The set of ordinary irreducible characters of H will be denoted by $\text{Irr}(H)$ and its elements will be referred to simply as irreducible characters. The set of (irreducible ℓ -) Brauer characters of H will be denoted by $\text{IBr}_\ell(H)$ or simply $\text{IBr}(H)$. The set $\text{Irr}(H)$ forms a basis of the space of class functions on H , which is endowed with its usual scalar product denoted by $\langle -, - \rangle$. The set $\text{IBr}(H)$ is a basis of the subspace of class functions on H vanishing outside the

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