Advances in Mathematics 307 (2017) 417–487

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim

Operad groups and their finiteness properties

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A R T I C L E   I N F O

Article history:
Received 18 December 2014
Received in revised form 24 June 2016
Accepted 14 November 2016
Available online xxxx
Communicated by Ross Street

MSC:
primary 20F65
secondary 57M07, 20F05, 18D50

Keywords:
Thompson groups
Operads
Finiteness properties

A B S T R A C T

We propose a new unifying framework for Thompson-like groups using a well-known device called operads and category theory as language. We discuss examples of operad groups which have appeared in the literature before. As a first application, we prove a theorem which implies that planar or symmetric or braided operads with transformations satisfying some finiteness conditions yield operad groups of type F_∞. This unifies and extends existing proofs that certain Thompson-like groups are of type F_∞.

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http://dx.doi.org/10.1016/j.aim.2016.11.022

1. Introduction

In unpublished notes of 1965, Richard Thompson defined three interesting groups F, T, V. For example, F is the group of all orientation preserving piecewise linear homeomorphisms of the unit interval with breakpoints lying in the dyadic rationals and with slopes being powers of 2. It has the presentation

$$F = \langle x_0, x_1, x_2, \dots \mid x_k^{-1} x_n x_k = x_{n+1} \text{ for } k < n \rangle.$$

In the subsequent years until the present days, hundreds of papers have been devoted to these and to related groups. The reason for this is that they have the ability to unite seemingly incompatible properties. For example, Thompson showed that V is an infinite finitely-presented simple group which contains every finite group as a subgroup. Even more is true: Brown showed in [7] that V is of type F_∞ which means that there is a classifying space for V with finitely many cells in every dimension. For F, this was proven by Brown and Geoghegan in [8]. They also showed that $H^k(F, \mathbb{Z}F) = 0$ for every $k \geq 0$. This implies in particular that all homotopy groups of F vanish at infinity and that F has infinite cohomological dimension. Thus, they found the first example of an infinite dimensional torsion-free group of type F_∞. In [6], Brin and Squier showed that F is a free group free group, i.e. contains no non-abelian free subgroups. Geoghegan conjectured in 1979 that F is non-amenable. If this is true, F would be an elegant counterexample to the von Neumann conjecture. Ol'shanskii disproved the von Neumann conjecture around 1980 by giving a different counterexample (see [30] and the references therein). Despite several attempts of various authors, the amenability question for F still seems to be open at the time of writing. During the 1970s, Thompson's group F was rediscovered twice: