



Contents lists available at ScienceDirect

## Advances in Mathematics

[www.elsevier.com/locate/aim](http://www.elsevier.com/locate/aim)


## Operad groups and their finiteness properties



Werner Thumann

Karlsruhe Institute of Technology, Karlsruhe, Germany

## ARTICLE INFO

*Article history:*

Received 18 December 2014  
 Received in revised form 24 June 2016  
 Accepted 14 November 2016  
 Available online xxxx  
 Communicated by Ross Street

*MSC:*

primary 20F65  
 secondary 57M07, 20F05, 18D50

*Keywords:*

Thompson groups  
 Operads  
 Finiteness properties

## ABSTRACT

We propose a new unifying framework for Thompson-like groups using a well-known device called operads and category theory as language. We discuss examples of operad groups which have appeared in the literature before. As a first application, we prove a theorem which implies that planar or symmetric or braided operads with transformations satisfying some finiteness conditions yield operad groups of type  $F_\infty$ . This unifies and extends existing proofs that certain Thompson-like groups are of type  $F_\infty$ .

© 2016 Elsevier Inc. All rights reserved.

## Contents

1.	Introduction	418
1.1.	Structure of the article	419
1.2.	Notation and conventions	421
1.3.	Acknowledgments	421
2.	Preliminaries on categories	421
2.1.	Comma categories	422
2.2.	The classifying space of a category	422
2.3.	The fundamental groupoid of a category	423
2.4.	Coverings of categories	424
2.5.	Contractibility and homotopy equivalences	425

E-mail address: [thumannw@gmail.com](mailto:thumannw@gmail.com).

<http://dx.doi.org/10.1016/j.aim.2016.11.022>

0001-8708/© 2016 Elsevier Inc. All rights reserved.

2.6.	Smashing isomorphisms in categories . . . . .	427
2.7.	Calculus of fractions and cancellation properties . . . . .	428
2.8.	Monoidal categories . . . . .	431
2.9.	Cones and joins . . . . .	433
2.10.	The Morse method for categories . . . . .	435
3.	Operad groups . . . . .	440
3.1.	Basic definitions . . . . .	440
3.2.	Normal forms . . . . .	445
3.3.	Calculus of fractions and cancellation properties . . . . .	446
3.4.	Operads with transformations . . . . .	447
3.5.	Examples . . . . .	451
4.	A topological finiteness result . . . . .	458
4.1.	Three types of arc complexes . . . . .	459
4.2.	A contractible complex . . . . .	467
4.3.	Isotropy groups . . . . .	467
4.4.	Finite type filtration . . . . .	471
4.5.	Connectivity of the filtration . . . . .	472
4.6.	Applications . . . . .	485
References	. . . . .	485

---

1. Introduction

In unpublished notes of 1965, Richard Thompson defined three interesting groups  $F, T, V$ . For example,  $F$  is the group of all orientation preserving piecewise linear homeomorphisms of the unit interval with breakpoints lying in the dyadic rationals and with slopes being powers of 2. It has the presentation

$$F = \langle x_0, x_1, x_2, \dots \mid x_k^{-1} x_n x_k = x_{n+1} \text{ for } k < n \rangle.$$

In the subsequent years until the present days, hundreds of papers have been devoted to these and to related groups. The reason for this is that they have the ability to unite seemingly incompatible properties. For example, Thompson showed that  $V$  is an infinite finitely-presented simple group which contains every finite group as a subgroup. Even more is true: Brown showed in [7] that  $V$  is of type  $F_\infty$  which means that there is a classifying space for  $V$  with finitely many cells in every dimension. For  $F$ , this was proven by Brown and Geoghegan in [8]. They also showed that  $H^k(F, \mathbb{Z}F) = 0$  for every  $k \geq 0$ . This implies in particular that all homotopy groups of  $F$  vanish at infinity and that  $F$  has infinite cohomological dimension. Thus, they found the first example of an infinite dimensional torsion-free group of type  $F_\infty$ . In [6], Brin and Squier showed that  $F$  is a free group free group, i.e. contains no non-abelian free subgroups. Geoghegan conjectured in 1979 that  $F$  is non-amenable. If this is true,  $F$  would be an elegant counterexample to the von Neumann conjecture. Ol’shanskii disproved the von Neumann conjecture around 1980 by giving a different counterexample (see [30] and the references therein). Despite several attempts of various authors, the amenability question for  $F$  still seems to be open at the time of writing. During the 1970s, Thompson’s group  $F$  was rediscovered twice:

Download English Version:

<https://daneshyari.com/en/article/5778686>

Download Persian Version:

<https://daneshyari.com/article/5778686>

[Daneshyari.com](https://daneshyari.com)