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Consistent systems of correlators in non-semisimple conformal field theory

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A B S T R A C T

Based on the modular functor associated with a – not necessarily semisimple – finite non-degenerate ribbon category $\mathcal{D}$, we present a definition of a consistent system of bulk field correlators for a conformal field theory which comprises invariance under mapping class group actions and compatibility with the sewing of surfaces. We show that when restricting to surfaces of genus zero such systems are in bijection with commutative symmetric Frobenius algebras in $\mathcal{D}$, while for surfaces of any genus they are in bijection with modular Frobenius algebras in $\mathcal{D}$. This provides additional insight into structures familiar from rational conformal field theories and extends them to rigid logarithmic conformal field theories.

1. Introduction and main result

A crucial task in any quantum field theory is to establish the existence of a consistent system of correlators of the fields of the theory. In two-dimensional conformal field

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theories these correlators are specific elements in suitable spaces of conformal blocks, characterized by the fact that they satisfy various consistency conditions. The spaces of conformal blocks of a conformal field theory can be explored from several different mathematical points of view. The approach relevant to the present paper describes them as finite-dimensional vector spaces that carry projective representations of mapping class groups of surfaces with marked points and are compatible with the sewing of surfaces. These vector spaces are constructed in terms of morphism spaces of a braided monoidal category $\mathcal{D}$ [3,37]. We refer to the data coming with the spaces of conformal blocks as the *monodromy data* based on $\mathcal{D}$ (they are also known as chiral data, or as Moore–Seiberg data).

Specifically, we consider local conformal field theories on closed oriented surfaces. For these, the fields are called bulk fields, and a consistent choice of bulk fields provides an object of $\mathcal{D}$, which we denote by F and to which for brevity we refer as the *bulk object*. In this paper we give a precise mathematical realization of the notions of bulk object and of systems of correlators of bulk fields for a conformal field theory corresponding to a given category $\mathcal{D}$. A novelty of our approach is that $\mathcal{D}$ does not have to be semisimple.

That the conformal block spaces are finite-dimensional is a non-trivial and useful finiteness property. Let us mention that this property is satisfied for the categories $\mathcal{D}$ that are relevant to interesting classes of conformal field theories, including in particular all rational conformal field theories, but also a large class of models for which $\mathcal{D}$ is non-semisimple. Because of the analytic properties of their conformal blocks, the latter models go under the name of *logarithmic* conformal field theories (see e.g. [25]). The fact that our approach does not require $\mathcal{D}$ to be semisimple thus makes it relevant for important applications of logarithmic conformal field theories like e.g. the study of critical dense polymers [9].

In this paper we develop a precise definition of the notion of consistency of a system of bulk field correlators (see Definition 3.16). Then we prove

Proposition 4.7 and Theorem 4.10. (i) *Let $\mathcal{D}$ be a finite ribbon category and F an object of $\mathcal{D}$. The consistent systems of genus-zero bulk field correlators for monodromy data based on $\mathcal{D}$ and with bulk object F are in bijection with structures of a commutative symmetric Frobenius algebra on F.*

(ii) *Let $\mathcal{D}$ be a modular finite ribbon category and F an object of $\mathcal{D}$. The consistent systems of bulk field correlators for monodromy data based on $\mathcal{D}$ and with bulk object F are in bijection with structures of a modular Frobenius algebra* (*in the sense of Definition* 4.9) *on F.*

Consistency conditions for correlators have been discussed extensively in the conformal field theory literature (see e.g. [16,34,44]). They amount to requiring that the correlator assigned to a surface is invariant under the action of the mapping class group of the surface and that upon sewing of surfaces, correlators are mapped to correlators [13]. (In addition, a non-degeneracy requirement must be imposed on the two-point correlator