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Higher equivariant excision

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We develop a theory of Goodwillie calculus for functors between G-equivariant homotopy theories, where G is a finite group. We construct J-excisive approximations for any finite G-set J. These combine into a poset, the Goodwillie tree, that extends the classical Goodwillie tower. We prove convergence results for the tree of a functor on pointed G-spaces that commutes with fixed-points, and we reinterpret the Tom Dieck-splitting as an instance of a more general splitting phenomenon that occurs for the fixed-points of the equivariant derivatives of these functors. As our main example we describe the layers of the tree of the identity functor in terms of the equivariant Spanier–Whitehead duals of the partition complexes.

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Introduction

Goodwillie calculus of functors was developed in the seminal papers [15–17] for functors between the categories of pointed spaces and spectra, and it was later extended to a model categorical framework in [4]. The key idea is to approximate a homotopy invariant functor between model categories $F\colon \mathscr{C} \to \mathscr{D}$ by a tower of weaker and weaker homology theories

$$F \to \big(\dots \longrightarrow P_nF \longrightarrow P_{n-1}F \longrightarrow \dots \longrightarrow P_1F\big).$$

This theory has found important applications in homotopy theory and in geometric topology (see for example [26,15,34,2]). If G is a finite group and $F\colon \mathscr{C}^G \to \mathscr{D}^G$ is a homotopy functor between model categories of G-objects (for example pointed G-spaces or genuine G-spectra) one can certainly apply Goodwillie's calculus machinery, but with a fundamental flaw. The Goodwillie tower above approximates F by “naïve” equivariant homology theories, as opposed to “genuine” ones. For example the first stage of the tower of the identity functor on pointed G-spaces is equivalent to

$$P_1I(X) \simeq \operatorname*{hocolim}_n \Omega^n\Sigma^n X$$

and not to the stabilization of X by the representations of G. The aim of this paper is to adapt Goodwillie calculus to a genuine equivariant context.

The central idea of equivariant calculus is to replace the cubes used in [17] to construct the Goodwillie tower with equivariant cubes that are indexed on finite G-sets. This is analogous to the way one replaces the natural numbers with the representations of G in the construction of genuine G-spectra. The paper is based on the foundations of homotopy theory of equivariant diagrams of [13], and particularly on the notion of cartesian and cocartesian equivariant cubes. A part of this material is recollected in §1.