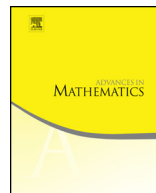




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Equivariant Moore spaces and the Dade group



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ABSTRACT

Let G be a finite p -group and k be a field of characteristic p . A topological space X is called an n -Moore space if its reduced homology is nonzero only in dimension n . We call a G -CW-complex X an $\underline{n}$ -Moore G -space over k if for every subgroup H of G , the fixed point set X^H is an $\underline{n}(H)$ -Moore space with coefficients in k , where $\underline{n}(H)$ is a function of H . We show that if X is a finite $\underline{n}$ -Moore G -space, then the reduced homology module of X is an endo-permutation kG -module generated by relative syzygies. A kG -module M is an endo-permutation module if $\text{End}_k(M) = M \otimes_k M^*$ is a permutation kG -module. We consider the Grothendieck group of finite Moore G -spaces $\mathcal{M}(G)$, with addition given by the join operation, and relate this group to the Dade group generated by relative syzygies.

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1. Introduction and statement of results

Let G be a finite group and M be a $\mathbb{Z}G$ -module. A G -CW-complex X is called a *Moore G -space* of type (M, n) if the reduced homology group $\tilde{H}_i(X; \mathbb{Z})$ is zero whenever $i \neq n$ and $\tilde{H}_n(X; \mathbb{Z}) \cong M$ as $\mathbb{Z}G$ -modules. One of the classical problems in algebraic topology, due to Steenrod, asks whether every $\mathbb{Z}G$ -module is realizable as the homology module of a Moore G -space. G. Carlsson [10] constructed counterexamples of non-realizable modules

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for finite groups that include $\mathbb{Z}/p \times \mathbb{Z}/p$ as a subgroup for some prime p . The question of finding a good algebraic characterization of realizable $\mathbb{Z}G$ -modules is still an open problem (see [21] and [4]).

In this paper we consider Moore G -spaces whose fixed point subspaces are also Moore spaces. Let R be a commutative ring of coefficients and let $\underline{n} : \text{Sub}(G) \rightarrow \mathbb{Z}$ denote a function from subgroups of G to integers, which is constant on the conjugacy classes of subgroups. Such functions are often called super class functions.

Definition 1.1. A G -CW-complex X is called an $\underline{n}$ -Moore G -space over R if for every $H \leq G$, the reduced homology group $\tilde{H}_i(X^H; R)$ is zero for all $i \neq \underline{n}(H)$.

When $\underline{n}$ is the constant function with value n for all $H \leq G$, the homology at dimension n can be considered as a module over the orbit category $\text{Or } G$. If $\tilde{H}_n(X^?; R) \cong \underline{M}$ as a module over the orbit category, then X is called a Moore G -space of type $(\underline{M}, n)$. When $R = \mathbb{Q}$ and X^H is simply-connected for all $H \leq G$, the space X is called a rational Moore G -space. Rational Moore G -spaces are studied extensively in equivariant homotopy theory and many interesting results are obtained on homotopy types of these spaces (see [16] and [11]).

In this paper, we allow $\underline{n}$ to be an arbitrary super class function and take the coefficient ring R as a field k of characteristic p . We define the group of finite Moore G -spaces over k and relate this group to the Dade group, the group of endo-permutation modules. Since the appropriate definition of a Dade group for a finite group is not clear yet, we restrict ourselves to the situation where G is a p -group, although the results have obvious consequences for finite groups via restriction to a Sylow p -subgroup.

Let G be a finite group and k be a field of characteristic p . Throughout we assume all kG -modules are finitely generated. A kG -module M is called an *endo-permutation kG -module* if $\text{End}_k(M) = M \otimes_k M^*$ is isomorphic to a permutation kG -module when regarded as a kG -module with diagonal G -action $(gf)(m) = gf(g^{-1}m)$. A G -CW-complex is called *finite* if it has finitely many cells. The main result of the paper is the following:

Theorem 1.2. *Let G be a finite p -group, and k be a field of characteristic p . If X is a finite $\underline{n}$ -Moore G -space over k , then the reduced homology module $\tilde{H}_n(X, k)$ in dimension $n = \underline{n}(1)$ is an endo-permutation kG -module generated by relative syzygies.*

This theorem is proved in Sections 3 and 4. We first prove it for tight Moore G -spaces (Proposition 3.8) and then extend it to the general case. An $\underline{n}$ -Moore space X is said to be *tight* if the topological dimension of X^H is equal to $\underline{n}(H)$ for every $H \leq G$. For tight Moore G -spaces, we give an explicit formula that expresses the equivalence class of the homology group $\tilde{H}_n(X, k)$ in terms of relative syzygies (see Proposition 3.8). This formula plays a key role for relating the group of Moore G -spaces to the group of Borel–Smith functions and to the Dade group. We now introduce these groups and the homomorphisms between them.

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