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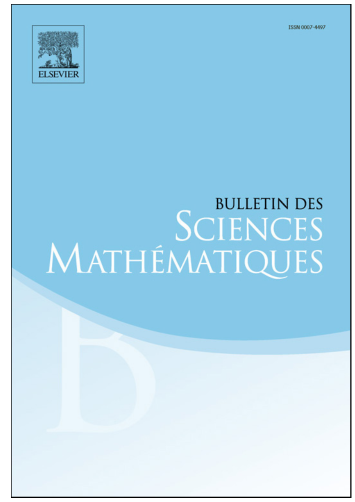

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ATOMIC DECOMPOSITION AND WEAK FACTORIZATION IN GENERALIZED HARDY SPACES OF CLOSED FORMS

ALINE BONAMI, JUSTIN FEUTO, SANDRINE GRELLIER, AND LUONG DANG KY

ABSTRACT. We give an atomic decomposition of closed forms on $\mathbb{R}^n$, the coefficients of which belong to some Hardy space of Musielak-Orlicz type. These spaces are natural generalizations of weighted Hardy-Orlicz spaces, when the Orlicz function depends on the space variable. One of them, called $\mathcal{H}^{\log}$, appears naturally when considering products of functions in the Hardy space $\mathcal{H}^1$ and in BMO. As a main consequence of the atomic decomposition, we obtain a weak factorization of closed forms whose coefficients are in $\mathcal{H}^{\log}$. Namely, a closed form in $\mathcal{H}^{\log}$ is the infinite sum of the wedge product between an exact form in the Hardy space $\mathcal{H}^1$ and an exact form in BMO. The converse result, which generalizes the classical div-curl lemma, is a consequence of [4]. As a corollary, we prove that the real-valued $\mathcal{H}^{\log}$ space can be weakly factorized.

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1. INTRODUCTION

Let φ be a $\mathcal{C}^\infty$ function with compact support on $\mathbb{R}^n$ such that $\int \varphi dx = 1$ and let φ_t denote the dilated function $\varphi_t(x) = t^{-n}\varphi(x/t)$. The Hardy space $\mathcal{H}^p(\mathbb{R}^n)$ is defined as the space of distributions f such that the function

$$f^+ := \sup_{t>0} |f * \varphi_t| \tag{1.1}$$

is in $L^p(\mathbb{R}^n)$. It is well-known, from the seminal work of Fefferman and Stein [10], not only that the definition does not depend on the particular function φ, but that the Hardy space $\mathcal{H}^p(\mathbb{R}^n)$ can be characterized in terms of the area or the grand maximal function. Its characterization in terms of the atomic decomposition for $0 < p \leq 1$ as initiated by Coifman in [7] when $n = 1$ and Latter in [16] when $n > 1$, revealed also a fundamental tool in the theory of Hardy spaces. Hardy spaces have been recently generalized in the context of Musielak-Orlicz spaces, first by the fourth author who proved the atomic decomposition [15], then by Dachun Yang et al. in [13, 17], where other equivalent properties are proved. For the definition of these new spaces, the Orlicz function t^p is replaced by a function $\wp(x,t)$, which belongs, as a function of x and uniformly in t, to the class of weights A_∞, while, as a function of t

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