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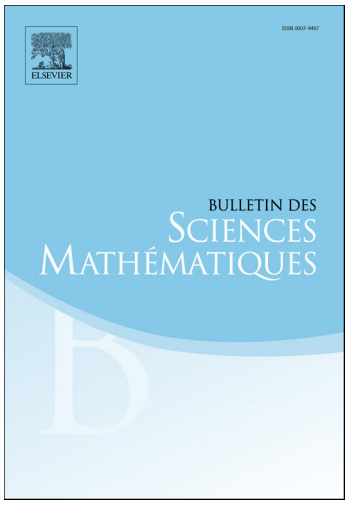

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Non-normal numbers: full Hausdorff dimensionality vs zero dimensionality[☆]

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Abstract

In the present paper we study the dependence of fractal and metric properties of numbers which are non-normal resp. essentialy non-normal w.r.t. a chosen system of numeration. In particular, we solve open problems mentioned in [1] and prove that there exist expansions (the Q^*-expansions or Q^*-representations) for real numbers such that the corresponding sets of essentially non-normal numbers and even the whole set of non-normal numbers are of **zero** Hausdorff dimension. On the other hand, we show that in the same model of Q^*-expansions it is possible to choose the matrix Q^* in such a way that the corresponding set of essentially non-normal numbers is of full Lebesgue measure. Sufficient conditions for full dimensionality resp. zero dimensionality of the set of essentially non-normal numbers are also presented.

Dans ce travail nous étudions la dépendance des propriétés fractales et métriques de nombres qui, par rapport à un système de numération donné, sont non-normales resp. essentiellement non-normales. En particulier nous don-

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