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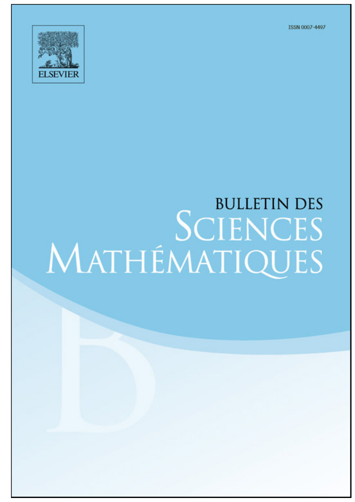

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Todd A. Oliynyk

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A PRIORI ESTIMATES FOR RELATIVISTIC LIQUID BODIES

TODD A. OLIYNYK

ABSTRACT. We demonstrate that a sufficiently smooth solution of the relativistic Euler equations that represents a dynamical compact liquid body, when expressed in Lagrangian coordinates, determines a solution to a system of non-linear wave equations with acoustic boundary conditions. Using this wave formulation, we prove that these solutions satisfy energy estimates without loss of derivatives. Importantly, our wave formulation does not require the liquid to be irrotational, and the energy estimates do not rely on divergence and curl type estimates employed in previous works.

1. INTRODUCTION

1.1. **Relativistic Euler equations.** On a 4-dimensional spacetime, the relativistic Euler equations are given by[1]

$$\nabla_\mu T^{\mu\nu} = 0 \tag{1.1}$$

where

$$T^{\mu\nu} = (\rho + p)v^\mu v^\nu + pg^{\mu\nu}$$

is the stress energy tensor,

$$g = g_{\mu\nu}dx^\mu dx^\nu$$

is a Lorentzian metric of signature $(-,+,+,+)$, ∇_μ is the Levi-Civita connection of $g_{\mu\nu}$, v^μ is the fluid four-velocity normalized by[2]

$$g_{\mu\nu}v^\mu v^\nu = -1,$$

ρ is the proper energy density of the fluid, and p is the pressure. Projecting (1.1) into the subspaces parallel and orthogonal to v^μ yields the following well known form of the relativistic Euler equations

$$v^\mu\nabla_\mu\rho + (\rho + p)\nabla_\mu v^\mu = 0, \tag{1.2}$$

$$(\rho + p)v^\mu\nabla_\mu v^\nu + h^{\mu\nu}\nabla_\mu p = 0, \tag{1.3}$$

where

$$h_{\mu\nu} = g_{\mu\nu} + v_\mu v_\nu \tag{1.4}$$

is the induced positive definite metric on the subspace orthogonal to v^μ. In this article, we will be concerned with fluids with a barotropic equation of state of the form

$$\rho = \rho(p)$$

where ρ satisfies

$$\rho \in C^\infty([0,\infty), [\rho_0, \rho_1]), \quad \rho(0) = \rho_0, \tag{1.5}$$

and

$$\rho'(p) > 0, \quad 0 \leq p < \infty, \tag{1.6}$$

for some constants $0 < \rho_0 < \rho_1$.

For fluid bodies with compact support, the timelike matter-vacuum boundary is defined by the vanishing of the pressure. Due to the above restrictions on the equation of state, the type of fluids considered in this article are *liquids*, which are characterized by having a jump discontinuity in the proper energy density at the matter-vacuum boundary. The main aim of this article is to derive a priori estimates for sufficiently smooth solutions of the relativistic Euler equations that represent dynamical compact liquid bodies. The precise form of the a priori estimates can be found in Theorem 8.1, which represent the main

[1]With the exception of Section 7, we use lower case Greek indices, i.e. μ, ν, γ, to label spacetime coordinate indices which run from 0 to 3.

[2]Following standard conventions, we lower and raise spacetime coordinate indices, i.e. μ, ν, γ, using the metric $g_{\mu\nu}$ and inverse metric $g^{\mu\nu}$, respectively.