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The diver with a rotor

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Abstract

We present and analyse a simple model for the twisting somersault. The model consists of a rigid body with a rotor attached that can be switched on and off. This makes it simple enough to devise explicit analytical formulas whilst still maintaining sufficient complexity to preserve the shape-changing dynamics essential for twisting somersaults performed in springboard and platform diving. With "rotor on" and with "rotor off" the corresponding Euler-type equations can be solved and the essential quantities characterising the dynamics, such as the periods and rotation numbers, can be computed in terms of complete elliptic integrals. We arrive at explicit formulas for how to achieve a dive with m somersaults and n twists in a given total time. This can be thought of as a special case of a geometric phase formula due to Cabrera (2007).

Keywords: Rigid body dynamics; Gyroscope; Twisting somersault; Geometric phase

1. Introduction

The analysis of the twisting somersault poses an interesting problem in classical mechanics. How can a body take off in pure somersaulting motion, initiate twisting midflight, and then return to pure somersaulting motion for entry into the water? Generally this is not a problem of rigid body dynamics, but instead of either non-rigid body dynamics or the description of coupled rigid

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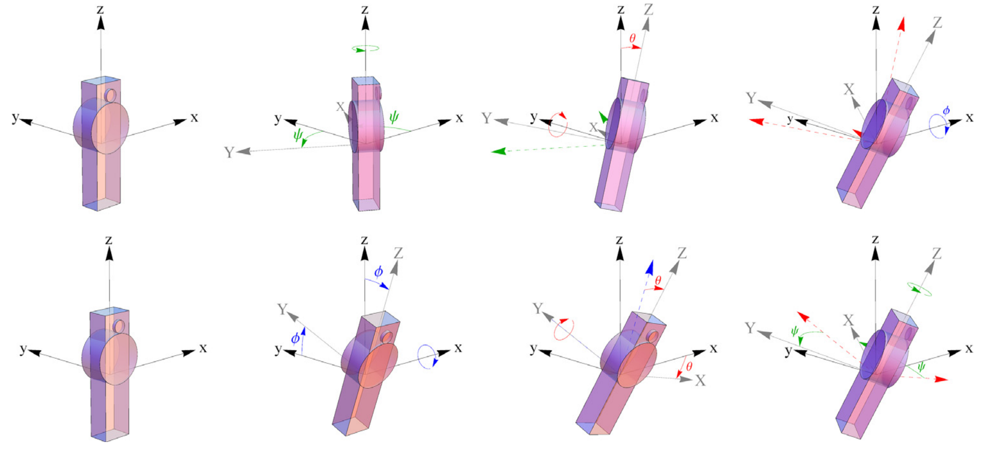

Fig. 1. A possible model of a rigid body (the box) and a disc that can be made to rotate about its symmetry axis. The box models the head, trunk, and legs of a human diver, while the disc models the arms, which can "rotate". In a robotic realisation the disc would actually rotate. The top line shows how the reference configuration is transformed to the final configuration by rotations about the initial axis. The bottom line shows how the same final configuration is reached by rotating about the intermediate axes in the reversed order.

bodies. Such a description of the twisting somersault was first proposed by [6,7] and has since been developed into a full-fledged analysis by Yeadon in a series of classical papers [14–18]. Here we are less ambitious in that we develop possibly the simplest model capable of exhibiting this kind of behaviour. The advantage of our model is that it is simple enough to be completely solved, thus allowing us to derive a precise equation that determines how exactly m somersaults and n twists can be performed in the total time T_{tot}, if at all. The model of the diver consists of a rigid body with a rotor attached. A rotation of the diver's arms is then simply modelled by switching the rotor on or off. The question we can answer is this: "When does the rotor need to be turned on to initiate twisting, and for how long should it stay on, off, and then on again to stop the twisting.?" From the dynamical systems point of view there are two autonomous systems ("rotor on" and "rotor off") that are switched between to achieve the desired trajectory. As such it is a discontinuous dynamical system whose solution is at least continuous. Despite its simplicity, the model appears to capture the essential features and even reasonable values of the parameters that are relevant in human springboard and platform diving. Whether we can learn something about human diving from this model – other than a rough idea of the fundamental principles – remains to be seen. However, we would like to propose that the simple device we are describing would make an interesting robot capable of performing twisting somersaults, potentially with many more twists than humanly possible.

2. Euler equations for a rigid body with a rotor

Let $\mathbf{l}$ be the constant angular momentum vector in a space fixed frame, and $\mathbf{L}$ the angular momentum vector in a reference frame moving with the body. Let R be the rotation matrix that transforms from one frame into the other, so that $\mathbf{l} = R\mathbf{L}$. The equations of motion for a rigid body with a rotor attached are well known, see e.g. [13,10,5,8,9]. Following Yeadon [16] we use an adapted system of Euler angles $R = R_1(\phi)R_2(\theta)R_3(\psi)$ where R_i is a rotation that fixes the ith axis, ϕ is the somersault angles, θ the tilt angle, and ψ the twist angle, see Fig. 1. This is the

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