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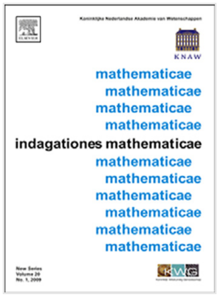

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Stability pockets of a periodically forced oscillator in a model for seasonality

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Abstract

A periodically forced oscillator in a model for seasonality shows chains of stability pockets in the parameter plane. The frequency of the oscillator and the length of the photoperiod in the Zeitgeber are the two parameters. The present study is intended as a theoretical complement to the numerical study of Schmal et al. [10] of stability pockets (or Arnol'd onions in their terminology). We construct the Poincaré map of the forced oscillator and show that the Arnol'd tongues are taken into chains of stability pockets by a map with a number of folds. This number is related to the rational point $(\frac{p}{q}, 0)$ on the frequency axis from which a chain of p pockets emanates. Stability pockets are already observed in an article by van der Pol & Strutt in 1928, see [13] and later explained by Broer & Levi in 1995, see [3].

Keywords: forced oscillator, circadian clock, entrainment, resonance, Poincaré map, stability pocket

1 Introduction

The numerical study by Schmal et al. [10] of a model for seasonal effects on the circadian clock of an organism (located in the supra chiasmatic nucleus), shows stability pockets and chains thereof in the parameter plane. As in [10] the seasonal effect is in particular the variation of the length of the daylight time interval, called photoperiod. This alternation of light and dark acts as a forcing, called Zeitgeber, on the circadian clock. The present study aims to complement the results of [10] by giving a theoretical background and moreover a geometrical explanation for the observed phenomena. We also indicate what is to be expected when the system is perturbed. The setting of the problem is bifurcations of parameter dependent dynamical systems, in particular periodically forced oscillators.

1.1 Periodically forced oscillators

We start with a general periodically forced oscillator. Such an oscillator shows periodic dynamics (also called synchronisation or entrainment) if the frequency of the forcing is close enough to a rational multiple of the frequency of the oscillator. The latter is the frequency of the isolated oscillator, that is in absence of forcing. On the other hand, if the frequency ratio is not close enough to a rational number, the dynamics is quasi periodic for a large number of irrational frequency ratio's. Now we consider the periodically forced oscillator as a system of two asymmetrically coupled oscillators. The first corresponds to the circadian clock and the second corresponds to the Zeitgeber, thus the first is forced by the second. The latter has fixed dynamics. In particular its frequency is fixed and we set it equal to one by a scaling of time. Thus the frequency of the first oscillator becomes a parameter of the system. Another important parameter is the coupling strength. In the parameter plane of coupling strength versus frequency there