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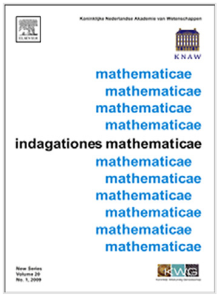

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SECTIONS OF UNIVALENT HARMONIC MAPPINGS

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Abstract. In this article, we determine the radius of univalence of sections of normalized univalent harmonic mappings for which the range is convex (resp. starlike, close-to-convex, convex in one direction). Our result on the radius of univalence of section $s_{n,n}(f)$ is sharp especially when the corresponding mappings have convex range. In this case, each section $s_{n,n}(f)$ is univalent in the disk of radius 1/4 for all $n \geq 2$, which may be compared with classical result of Szegö on conformal mappings.

1. Introduction and main results

Since confirmation of the Bieberbach conjecture by Louis de Branges [3] on the class $\mathcal{S}$, of all normalized univalent analytic functions ϕ defined in the unit disk $\mathbb{D} = \{z : |z| < 1\}$, one of the open problems about the class $\mathcal{S}$ is that of determining the precise value of r_n such that all sections $s_n(\phi)$ of ϕ are univalent in $|z| < r_n$. Here we say that ϕ is normalized if $\phi(0) = 0 = \phi'(0) - 1$. Also, let

$$s_n(\phi)(z) = \sum_{k=1}^{n} a_k z^k \tag{1}$$

whenever

$$\phi(z) = \sum_{k=1}^{\infty} a_k z^k. \tag{2}$$

In [24], Szegö proved that the section/partial sum $s_n(\phi)$ of $\phi \in \mathcal{S}$ is univalent in $|z| < 1/4$ for all $n \geq 2$. The constant 1/4 is sharp as the second section of the Koebe function $k(z) = z/(1-z)^2$ suggests. In [20], Robertson proved that the section $s_n(k)$ is starlike in the disk $|z| < 1 - 3n^{-1} \log n$ for $n \geq 5$, and that the number 3 cannot be replaced by a smaller constant. Later in the year 1991, Bshouty and Hengartner [4] showed that the Koebe function is not extremal for the problem of determining the radius of univalency of the partial sums of functions in $\mathcal{S}$. At this time the best known result is due to Jenkins [11] who proved that $s_n(\phi)$ is univalent in $|z| < r_n$ for $\phi \in \mathcal{S}$, where the radius of univalence r_n is at least $1 - (4 \log n - \log(4 \log n))/n$ for $n \geq 8$. For related investigations on this topic, see the recent articles [14, 16] and the references therein. More interestingly, as investigated recently in [12, 13], our

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