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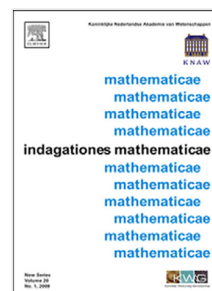
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## SECTIONS OF UNIVALENT HARMONIC MAPPINGS

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ABSTRACT. In this article, we determine the radius of univalence of sections of normalized univalent harmonic mappings for which the range is convex (resp. starlike, close-to-convex, convex in one direction). Our result on the radius of univalence of section  $s_{n,n}(f)$  is sharp especially when the corresponding mappings have convex range. In this case, each section  $s_{n,n}(f)$  is univalent in the disk of radius  $1/4$  for all  $n \geq 2$ , which may be compared with classical result of Szegő on conformal mappings.

## 1. INTRODUCTION AND MAIN RESULTS

Since confirmation of the Bieberbach conjecture by Louis de Branges [3] on the class  $\mathcal{S}$ , of all normalized univalent analytic functions  $\phi$  defined in the unit disk  $\mathbb{D} = \{z : |z| < 1\}$ , one of the open problems about the class  $\mathcal{S}$  is that of determining the precise value of  $r_n$  such that all sections  $s_n(\phi)$  of  $\phi$  are univalent in  $|z| < r_n$ . Here we say that  $\phi$  is normalized if  $\phi(0) = 0 = \phi'(0) - 1$ . Also, let

$$(1) \quad s_n(\phi)(z) = \sum_{k=1}^n a_k z^k$$

whenever

$$(2) \quad \phi(z) = \sum_{k=1}^{\infty} a_k z^k.$$

In [24], Szegő proved that the section/partial sum  $s_n(\phi)$  of  $\phi \in \mathcal{S}$  is univalent in  $|z| < 1/4$  for all  $n \geq 2$ . The constant  $1/4$  is sharp as the second section of the Koebe function  $k(z) = z/(1-z)^2$  suggests. In [20], Robertson proved that the section  $s_n(k)$  is starlike in the disk  $|z| < 1 - 3n^{-1} \log n$  for  $n \geq 5$ , and that the number 3 cannot be replaced by a smaller constant. Later in the year 1991, Bshouty and Hengartner [4] showed that the Koebe function is not extremal for the problem of determining the radius of univalence of the partial sums of functions in  $\mathcal{S}$ . At this time the best known result is due to Jenkins [11] who proved that  $s_n(\phi)$  is univalent in  $|z| < r_n$  for  $\phi \in \mathcal{S}$ , where the radius of univalence  $r_n$  is at least  $1 - (4 \log n - \log(4 \log n))/n$  for  $n \geq 8$ . For related investigations on this topic, see the recent articles [14, 16] and the references therein. More interestingly, as investigated recently in [12, 13], our

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