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Quasi-compact operator, pseudo-essential spectra and some perturbation results

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Abstract

In this paper, we use the concept of quasi-compact operators, as a generalization of the class of Riesz operators, to improve the definition of the pseudo-Schechter essential spectrum of a closed densely defined operator acting on Banach space. Moreover, we discuss the incidence of some perturbation results on the behavior of pseudo-essential spectra of the sum of two bounded linear operators.

Keywords: Quasi-compact operator; Pseudo-essential spectra; Pseudo Fredholm inverse

1. Introduction

Throughout this paper, X will be a complex infinite dimensional Banach space. We denote by $\mathcal{L}(X)$ (resp. $\mathcal{C}(X)$) the set of all bounded (resp. closed, densely defined) linear operators on X. The subset of all compact of $\mathcal{L}(X)$ is designated by $\mathcal{K}(X)$. For $A \in \mathcal{C}(X)$, we write $\mathcal{D}(A)$ for the domain, $\rho(A)$ (resp. $\sigma(A)$) for the resolvent set (resp. the spectrum) of A, $N(A)$ for the null space and $R(A)$ for the range of A.

The nullity, $\alpha(A)$, of A is defined as the dimension of $N(A)$ and the deficiency, $\beta(A)$, of A is defined as the codimension of $R(A)$ in X. The set of upper semi-Fredholm operators from X

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into Y is defined by:

$$\Phi_+(X) := \{A \in \mathcal{L}(X) : \alpha(A) < \infty \text{ and } R(A) \text{ is closed in } X\},$$

the set of lower semi-Fredholm operators on X is defined by

$$\Phi_-(X) := \{A \in \mathcal{L}(X) : \beta(A) < \infty \text{ and } R(A) \text{ is closed in } X\}.$$

$\Phi_\pm(X) := \Phi_+(X) \cup \Phi_-(X)$ denotes the set of semi-Fredholm operators on X and $\Phi(X) := \Phi_+(X) \cap \Phi_-(X)$ denotes the set of Fredholm operators on X. The intersections, $\Phi(X) \cap \mathcal{L}(X)$, $\Phi_+(X) \cap \mathcal{L}(X)$, $\Phi_-(X) \cap \mathcal{L}(X)$, are denoted by $\Phi^b(X)$, $\Phi^b_+(X)$, $\Phi^b_-(X)$. Let

$$\Phi_A := \{\lambda \in \mathbb{C} : \lambda - A \in \Phi(X)\}.$$

For $A \in \Phi(X)$, the number $i(A) = \alpha(A) - \beta(A)$ is called the index of A.

We denote by $\mathcal{R}(X)$ the class of all Riesz operators which is characterized in [1] by:

$$\mathcal{R}(X) := \left\{A \in \mathcal{L}(X) : \lambda - A \in \Phi(X) \text{ for each } \lambda \neq 0\right\}.$$

Let $\sigma(A)$ (resp. $\rho(A)$) denote the spectrum (resp. the resolvent set) of A. For every $\varepsilon > 0$, the pseudo-spectrum of a densely closed linear operator A is defined as:

$$\sigma_\varepsilon(A) := \sigma(A) \cup \left\{\lambda \in \mathbb{C} : \|(A - \lambda)^{-1}\| > \frac{1}{\varepsilon}\right\}.$$

The pseudo-spectrum is the open subset of the complex plane bounded by the ε^{-1} level curve of the norm of the resolvent.

Inspired by the notion of pseudo-spectra, A. Ammar and A. Jeribi in their works [2–4,6], thought to extend these results for the essential spectra of closed, densely defined, and linear operators on a Banach space. They declared the new concept of the pseudo-essential spectra of closed, densely defined, and linear operators on a Banach space. More precisely, for $A \in \mathcal{C}(X)$ and for every $\varepsilon > 0$, they defined the pseudo-Schechter and the pseudo-Browder essential spectrum in the following way:

$$\sigma_{e5,\varepsilon}(A) := \bigcap_{K \in \mathcal{K}(X)} \sigma_\varepsilon(A + K),$$

$$\sigma_{e6,\varepsilon}(A) := \sigma_{e6}(A) \cup \left\{\lambda \in \mathbb{C} : \|R_b(A, \lambda)\| > \frac{1}{\varepsilon}\right\}$$

where $\sigma_{e6}(A) := \mathbb{C} \setminus \rho_6(A)$ with

$$\rho_6 := \{\lambda \in \Phi_A : i(\lambda - A) = 0 \text{ and all scalars near } \lambda \text{ are in } \rho(A)\}$$

and $R_b(A, \lambda) = ((A - \lambda)\,|_{K_\lambda})^{-1}(I - P_\lambda) + P_\lambda$ where P_λ the Riesz projection corresponding to λ and K_λ is the kernel of P_λ.

Let $A \in \mathcal{C}(X)$, we know that $\mathcal{D}(A)$ provided with the graph norm $\|x\|_A = \|x\| + \|Ax\|$ is a Banach space denoted by X_A. In this new space, the operator A satisfies $\|Ax\| \leq \|x\|_A$ and consequently A is a bounded operator from X_A into X. Recall that an operator B is A-bounded if $\mathcal{D}(A) \subseteq \mathcal{D}(B)$ and B is bounded on X_A. For $A \in \mathcal{C}(X)$, let B be an arbitrary A-bounded operator on X. Hence, we can regard A and B as bounded operators from X_A into X. They will be denoted by $\hat{A}$ and $\hat{B}$, respectively. These belong to $\mathcal{L}(X_A, X)$. Furthermore, we have the

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