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Generalization of some hypergeometric functions

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Abstract

In this paper, we make use of the Lagrange interpolation to obtain some algebraic identities, involving one or two infinite sets of variables.

Keywords: Symmetric functions; Divided differences; Lagrange interpolation

1. Introduction

One of the most important general expansion formulas for hypergeometric series is the Fields and Wimp expansion, described by [5]:

$$
{}_{r+t}F_{s+u}\left[\begin{matrix} a_R, & c_T \\ b_S, & d_U \end{matrix}; xw\right] = \sum_{n=0}^{\infty} \frac{(a_R)_n(\alpha)_n(\beta)_n}{(b_S)(\gamma+n)_n} \frac{(-x)^n}{n!} {}_{r+2}F_{s+1} \\
\times \left[\begin{matrix} n+\alpha, & n+\beta, & n+a_R \\ 1+2n+\gamma, & n+b_S & \end{matrix}; x\right]
$$

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$$\times\, {}_{t+2}F_{u+2}\left[\begin{matrix}-n, & n+\gamma, & c_T \\ \alpha, & \beta & d_U\end{matrix}; w\right],$$

where the contract notations $(a)_n$, a_R, $(a_R)_n$, and $n+a_R$ represent $a(a+1)\ldots(a+n-1)$, $a_1,\ldots,a_r$, $(a_R)_n$, $(a_1)_n\,(a_2)_n\ldots(a_r)_n$, and $n+a_1,\ldots,n+a_r$, respectively.

Verma showed in [8] that this formula is a special case of expansion (6.1) and derived the q-analog (6.2).

On the other hand, Al-Salam and Verma [3] showed that Euler's transformation formula

$$\sum_{n=0}^{\infty} a_n b_n x^n = \sum_{k=0}^{\infty} (-1)^k \frac{x^k}{k!} f^{(k)}(x) \Delta^k a_0, \tag{1.1}$$

where

$$f(x) = b_0 + b_1 x + b_2 x^2 + \cdots,$$

and

$$\Delta^k a_0 = \sum_{j=0}^{k} (-1)^j \binom{k}{j} a_{k-j},$$

has bibasic extension (5.2).

For their part, Gessel and Stanton [6] obtained the following generating function

$$\frac{(1+x)^a(1+y)^b}{(1-xy)^{a+b+1}} = \sum_{k=0,\,p=0}^{\infty} \binom{a+p}{k}\binom{b+k}{p} x^k y^p. \tag{1.2}$$

In this paper, we make use of the divided differences [2] and the Lagrange interpolation to obtain algebraic identities, involving one or two infinite sets of variables (formulas (4.1)–(4.3)). By specializing variables sets, we recover formulas ((6.1) and (6.2)) provided by Verma [8], formula (5.2) given by Al-Salem and Verma [3], and generating function (7.1) of Gessel and Stanton [6].

We also give a new q-analog of results ((5.3), (5.4), (6.3), (6.4), (7.3), (7.4), (7.5)).

For this purpose, the Lagrange interpolation is considered to describe the properties of a linear operator, sending function of one variable to a symmetric function [4]. It can be written as a summation on a set, or a product of divided differences. This later version will be used throughout this paper.

2. Multiple interpolations

In this section, we consider the linear operator $\Lambda(A)$ of the Lagrange interpolation defined by

$$\Lambda(A)(f) = \sum_{a \in A} \frac{f(a)}{R(a, A \setminus a)} \tag{2.1}$$

where $R(a,B) = R_{b\in B}(a-b)$ and $R(a,\emptyset) = 1$.

Accordingly, this operator sends a polynomial of degree k to a symmetric polynomial in A of degree $k-n$, with $card(A) = n+1$. In particular, it annihilates polynomials of degree $< n$, and $f(x) = x^n$ on constant 1.