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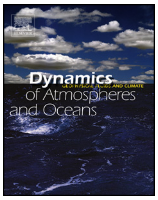

Short communication

A note on free and forced Rossby wave solutions: The case of a straight coast and a channel

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ABSTRACT

The free Rossby wave (RW) solutions in an ocean with a straight coast when the offshore wavenumber of incident (l_1) and reflected (l_2) wave are equal or complex are discussed. If $l_1 = l_2$ the energy streams along the coast and a uniformly valid solution cannot be found; if $l_{1,2}$ are complex it yields the sum of an exponentially decaying and growing (away from the coast) Rossby wave. The channel does not admit these solutions as free modes.

If the wavenumber vectors of the RWs are perpendicular to the coast, the boundary condition of no normal flow is trivially satisfied and the value of the streamfunction does not need to vanish at the coast. A solution that satisfies Kelvin's theorem of time-independent circulation at the coast is proposed.

The forced RW solutions when the ocean's forcing is a single Fourier component are studied. If the forcing is resonant, i.e. a free Rossby wave (RW), the linear response will depend critically on whether the wave carries energy perpendicular to the channel or not. In the first case, the amplitude of the response is linear in the direction normal to the channel, y, and in the second it has a parabolic profile in y. Examples of these solutions are shown for channels with parameters resembling the Mozambique Channel, the Tasman Sea, the Denmark Strait and the English Channel. The solutions for the single coast are unbounded, except when the forcing is a RW trapped against the coast. If the forcing is non-resonant, exponentially decaying or trapped RWs could be excited in the coast and both the exponentially "decaying" and exponentially "growing" RW could be excited in the channel.

1. Introduction

In an unforced and inviscid ocean with an arbitrarily oriented straight coast there are three linear Rossby mode solutions. First, there is the well known solution representing reflection of RWs (Longuet-Higgins, 1964, 1965; Pedlosky, 2013). The second one is when the offshore wavenumbers of incident and reflected wave, $l_{1,2}$, are equal, in which case there is no reflection and the solution is a single wave whose amplitude grows linearly in the offshore direction. Finally when $l_{1,2}$ are complex, the solution is a sum of an exponentially decaying, or trapped RW against the coast, and an exponentially growing (from the coast) RW.

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The second and third solutions are discussed in the author's PhD dissertation Graef-Ziehl (1990) and mentioned in Graef (1993) and Graef and Magaard (1994) in the context of the reflection problem of weakly nonlinear RWs, i.e. for the case of a single coast. Also García and Graef (1998) used them when these authors studied the nonlinear self-interaction of a Rossby mode in a channel.

However, several questions that could be of interest from the Geophysical Fluid Dynamics (GFD) point of view remained open:

- If the wavenumber vectors of the RWs are perpendicular to the coast, the boundary condition of no normal flow at the coast is trivially satisfied. Is there a solution that satisfies Kelvin's theorem of time-independent circulation (Pedlosky, 2013) at the coast?
- Is it possible to find a uniformly valid solution if $l_1 = l_2$?
- Are these solutions excited in a more general forced problem? That is, the forcing in this paper does not have anything to do with the particular forcing that results from the weakly nonlinear interaction of an incident and the reflected RW or from the self-interaction of a channel Rossby mode. Here there is no perturbation expansion and the governing equation is the linear QG potential vorticity equation.
- If the forcing is resonant, what is the linear response?
- Does the forced solution depend on whether the forcing RW carries energy perpendicular to the coast or channel?

In this note I provide an answer to these questions, thus advancing our knowledge in GFD. Indeed, a new solution that has time-independent circulation (actually zero circulation) is proposed. The method of multiple scales is used in an attempt to find a uniformly valid solution when $l_1 = l_2$. When a more general forcing (in the form of a single Fourier component) of the linear QG potential vorticity equation is considered, the linearly growing and exponentially decaying or growing solutions do get excited. The most interesting case is when the forcing is resonant, i.e. a free RW. It turns out that the linear response will depend critically on whether the wave carries energy perpendicular to the coast (or channel) or not. If the forcing is non-resonant (not a free RW), in the form of a Fourier component, then the exponentially decaying RW could be excited in the coast and both the exponentially "decaying" and exponentially "growing" RW could be excited in the channel. It is worth noting that the excitation of solutions that might otherwise be overlooked illustrates a general principle: it is always important to know *all* available solutions of a linear and homogeneous problem.

In the channel, the analogous case of the wavenumber vectors perpendicular to the coast corresponds to modes or solutions that are independent of the coordinate along the channel. These modes were studied extensively by Graef and Müller (1996); later LaCasce (2000) and LaCasce and Pedlosky (2002) (for a meridional channel in their one dimensional case) reproduced the solutions of Graef and Müller (1996) without citing them.

Beyond the value of increasing our knowledge in GFD, this work is also motivated by some oceanographic and numerical studies of RWs in channels and coasts. If one looks at the world's oceans, the most conspicuous mid-latitude channels for which planetary wave motion could matter are the Mozambique Channel, the Tasman Sea, the Denmark Strait and perhaps (because of their irregularity and/or size) the South China Sea, the Caribbean Sea, and the English Channel. A Mozambique channel Rossby normal mode was inferred from flow observations by Harlander et al. (2009). On the numerical side, Marchesiello and Middleton (2000) pointed out that the RW reflection is key in modelling the East Australian Current. As regards to the Denmark Strait, only topographic RWs seem to be important (see for example von Appen et al., 2014). Regarding the South China Sea, Lin et al. (2016) observed annual RWs from a temperature mooring, whereas observations of currents and sea level carried out by Shu et al. (2016), Yang and Liu (2003), Wu et al. (2008) were interpreted as free and forced RWs. Concerning the English Channel, Webb (2013) investigated the shelf resonances and showed numerically that continental shelf/RWs have a significant effect on the diurnal tide. These are just a few examples illustrating the importance of RWs to explain observations and interpret modelling efforts.

In the next section the free Rossby mode solutions in an ocean with a straight coast are discussed, emphasizing the cases when the offshore wavenumbers are equal or complex and illustrating them graphically in relation to the RW dispersion relation. One subsection is devoted to the solutions that are independent of the coordinate along the coast and have zero circulation whereas other subsection briefly mentions the channel modes. The forced Rossby wave solutions for a single coast and a channel are studied in Section 3, for both resonant and non-resonant forcing. To put this work in an oceanographic context, some examples of the solutions with parameters resembling real channels like the Mozambique Channel, the Tasman Sea, the Denmark Strait and the English Channel are shown. Finally, the last section is devoted to a summary of the results and conclusions.

2. Free Rossby modes

Consider a β-plane with a coordinate system (x, y, z) in which x is parallel, y perpendicular to the coast or wall and z vertically upwards (Fig. 1). There is a vertical wall at the plane $y = 0$. The origin is somewhere in a mid-latitude region. The governing equation is the linear QG potential vorticity equation (QGPVE), which in this coordinate system reads

$$\mathcal{L}\psi \equiv \left\{\partial_t \left[\nabla^2 + \partial_z(Z^2\partial_z)\right] + \beta(\cos\alpha\,\partial_x + \sin\alpha\,\partial_y)\right\}\psi = 0, \qquad (1)$$