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A new model of marine sediment compression

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ABSTRACT

Marine sediments cover two-thirds of the earth, and porosity (or void ratio) is a major controlling parameter in virtually every model of seafloor properties, including strength, sound speed, hydrology, thermal conductivity, and electrical resistivity. Our new model of void ratio (e) is based on the proportional void ratio, $[e_\mathrm{p} = (e - e_\mathrm{r})/(e_0 - e_\mathrm{r})]$, where e_0 is the depositional maximum at the sea floor, and e_r is the minimum residual void ratio at depth. We assume the values of e_0 and e_r are inherent characteristics of the sediment type. Our model further defines the compression index C_c to be the square root of the proportional void ratio ($C_\mathrm{c}(e) = (e_\mathrm{p})^{1/2}$). This new formulation establishes a direct relation between void ratio and effective stress: $e = (e_0 - e_\mathrm{r})^{-1}[\log_{10}(\sigma_0/\sigma) + 2(e_0 - e_\mathrm{r})]^2/4 + e_\mathrm{r}$ and exhibits several advantages over previous models that we demonstrate with compression test data from the Gulf of Mexico and Nankai Trough.

1. Introduction

In order to understand sub-seafloor processes relevant to resource exploration, fluid cycling, slope stability and hazard analysis, scientists must first model the physical properties of the sediment column. Physical models of sub-seafloor properties, including density, sound speed, thermal conductivity, electrical resistivity and others, depend significantly on porosity (or equivalently, void ratio). Knowing the stress–strain relationship for each layer in a column of sediment allows estimation of the porosity from deposition at the seafloor to deep burial. From porosity, one can then estimate numerous sediment geophysical parameters and implement existing models, such as the sediment physics model of Dvorkin et al. (1999), the thermal property models of Goto and Matsubayashi (2009), and Waite et al. (2009), the permeability model of Revil and Cathles (1999) and the resistivity relationships of Archie (1942) and Collett and Ladd (2000), to create an extensive physical model of the sediment.

We present here a new formulation for the reduction of sediment porosity (sediment compression) with increasing effective stress. The result is an equation relating void ratio (e) to the $\log_{10}$ of the vertical effective stress (σ'), which can be used to model void ratio as a function of depth. From the void ratio, and other inputs, we could use the models mentioned above to estimate pressure, temperature, density, sound speed, shear wave speed, and electrical resistivity. Because these models have roots in both marine geophysical and geotechnical literature, we use in this paper both porosity (ϕ), and void ratio (e), to describe the volume fraction of void space, where $e = \phi/(1-\phi)$, and $\phi = e/(1+e)$.

The expression for the compression of marine sediment during normal consolidation used in recent literature (e.g., Long et al., 2011; Dugan, 2012) dates back to Terzaghi and Peck (1948) and is based on the change in void ratio being proportional to the base 10 logarithm of the vertical effective stress:

$$e_2 = e_1 - C_\mathrm{c} \log_{10}(\sigma') \tag{1}$$

where C_c is the constant of proportionality, also called the compression index, and e_1 and e_2 are the initial and final void ratios. The vertical effective stress, σ' (in kPa), is the load supported by the grains, equal to the difference between the total lithostatic pressure (σ) and the pore pressure. In Eq. (1), C_c is an empirical constant, equal to the slope of the $e - \log_{10}(\sigma')$ curve for a given sediment type. Since C_c is assumed to be constant, the virgin consolidation curve in $e - \log_{10}(\sigma')$ space is modeled as a straight line. However, it has been observed for some time (e.g., Butterfield, 1979) that the stress–strain curve, or $e - \log_{10}(\sigma')$ curve, is concave upward (as in Fig. 1), implying that C_c is not constant, but actually a function of void ratio (Long et al., 2011). The variation in C_c with void ratio is particularly notable in clay-rich marine sediments where void ratio varies over a larger range than it does in sand.

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Nomenclature

ϕ	Porosity: pore volume/total volume $= e/(1+e)$
e	Void ratio: pore volume/solid volume $= \phi/(1-\phi)$
σ	Total stress: assumed to be lithostatic pressure
σ'	Vertical effective stress: load supported by the grains (difference between lithostatic and pore pressure)
σ_0, e_0, ϕ_0	Initial stress, void ratio, porosity: depositional values (at the seafloor)
σ_r, e_r, ϕ_r	Residual stress, void ratio, porosity: values at maximum compression (minimum pore volume) without crushing sediment grains
ϕ_p	Proportional Porosity $= (\phi - \phi_r)/(\phi_0 - \phi_r)$
e_p	Proportional Void Ratio $= (e - e_r)/(e_0 - e_r)$
C_c	Compression Index (e.g., Long et al., 2011)
v	Specific Volume (Butterfield, 1979)
g	$= \log_{10}(\sigma)$
a	$= 1.0/(e_0 - e_r)$
b	$= -e_r/(e_0 - e_r)$

In order to build the best possible model of sediment parameters, the modeled $e - \log_{10}(\sigma')$ curve should fit observed data as closely as possible at all stresses. Therefore, the modeled relationship must be concave upward, and produce only porosities which are physically possible. Butterfield (1979) achieved a concave upward $e - \log_{10}(\sigma')$ curve by setting specific volume ($v = 1 + e$) proportional to change in effective stress, producing the equation $v = v_0(\sigma')^C$, where v_0 is the initial void ratio, and C is a fitting parameter. Long et al. (2011) showed that this equation fit consolidation tests on samples from IODP Sites U1322 and U1324 in the Gulf of Mexico better than the traditional geomechanical models (e.g., Eq. (1)) because it better mimicked the concave upward behavior of the $e - \log_{10}(\sigma')$ curve. However, the Butterfield (1979) relationship still produces negative porosities at high effective stresses. Since porosity cannot reach negative values, the curvature of the true stress–strain curve must be fundamentally different from that represented by the Butterfield equation.

2. Model formulation

Our goal is to derive a better model for the virgin consolidation curve by allowing C_c to vary with void ratio. Long et al. (2011) suggest a linear relationship between C_c and void ratio (e). Such a relationship can theoretically produce negative porosities, which we wish to avoid. To determine a better functional form for C_c, we first make some of the same assumptions as these previous models: that the dominant method of porosity reduction due to increased vertical effective stress is by the re-arrangement of grains through rotation, sliding, and bending with no significant mineral precipitation, crushing, or melting. We define the depositional porosity, ϕ_0, as the porosity at which the sediment falls out of suspension, which Dvorkin et al. (1999) refers to as the critical porosity. The vertical effective stress at ϕ_0 is σ_0, which is not equal to zero, but is very small. The particular value of σ_0 is important for determining the starting point of a stress–strain curve, and is discussed in Section 3.2.

We define ϕ_r as the residual porosity, which is the porosity at which the grains have been re-arranged to their maximum packing efficiency. Beyond this limit, the dominant method of porosity reduction is by crushing, melting, or chemical alteration of individual grains. We then define the proportional porosity, ϕ_p, as the fraction of the way from ϕ_0 to ϕ_r, or equivalently for proportional void ratio, e_p, from e_0 to e_r;

$$\phi_p = (\phi - \phi_r)/(\phi_0 - \phi_r), \qquad e_p = (e - e_r)/(e_0 - e_r).$$

As our results will show, we have found empirically that the compression index is extremely well represented by the square root of the proportional void ratio;

$$C_c(e) = (e_p)^{1/2} = \left[(e - e_r)/(e_0 - e_r)\right]^{1/2}. \tag{2}$$

We offer no physical justification for this functional form; it is entirely empirical. However, it offers a significant advantage over previous forms in that it is everywhere geologically reasonable. Neither negative nor infinite void ratios are ever encountered for any stress. (Equivalently, no porosity is ever less than 0 or greater than 100%.) Eq. (2) now gives us an expression for the slope at any point along the stress–strain curve.

Letting $g = \log_{10}(\sigma')$, and putting Eq. (1) in differential form yields

$$de = -C_c(e)dg = -\left[(e - e_r)/(e_0 - e_r)\right]^{1/2} dg$$

Substituting $a = 1.0/(e_0 - e_r)$ and $b = -e_r/(e_0 - e_r)$ produces

$$de = -(ae + b)^{1/2} dg.$$

Accumulating terms of "e" on one side yields

$$dg = -(ae + b)^{-1/2} de.$$

Integrating both sides yields

$$g = -2(ae + b)^{1/2}/a + C,$$

where C is the constant of integration. To determine a value for C, we apply a boundary condition: at minimum void ratio $e = e_r$, making $ae_r + b = 0$. Therefore, $C = g(e_r) = g_r = \log_{10}(\sigma_r')$. In other words, the constant of integration, C, is the $\log_{10}$ of the vertical effective stress required to reach the minimum void ratio. Substituting g_r for C,

$$g = g_r - 2(ae + b)^{1/2}/a$$

or

$$\log_{10}(\sigma') = \log_{10}(\sigma_r) - 2(ae + b)^{1/2}/a \tag{3a}$$

$$e = a^{-1}\left[\left(a \log_{10}(\sigma_r/\sigma')/2\right)^2 - b\right] \tag{3b}$$

$$e = \frac{1}{4(e_0 - e_r)} \log_{10}^2\left(\frac{\sigma_r}{\sigma}\right) + e_r \tag{3c}$$

The initial condition of the system is at maximum void ratio ($e = e_0$), where $ae_0 + b = 1$, $g_0 = g_r - 2/a$, and therefore

$$e = \frac{1}{4(e_0 - e_r)} \left[\log_{10}\left(\frac{\sigma_0}{\sigma}\right) + 2(e_0 - e_r)\right]^2 + e_r \tag{4a}$$

Expanding and simplifying yields

$$e = \frac{1}{4(e_0 - e_r)} \log_{10}^2\left(\frac{\sigma_0}{\sigma}\right) + \log_{10}\left(\frac{\sigma_0}{\sigma}\right) + e_0 \tag{4b}$$

Eqs. (3) and (4) are closed form expressions for vertical effective stress as a function of void ratio and vice versa. Only three parameters are required to span the spaces described in these equations, namely e_0, e_r and either σ_0 or σ_r. Relating e_0 to σ_0 will reduce the necessary parameters to two.