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Research paper

Population synthesis of planet formation using a torque formula with dynamic effects

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ABSTRACT

Population synthesis studies into planet formation have suggested that distributions consistent with observations can only be reproduced if the actual Type I migration timescale is at least an order of magnitude longer than that deduced from linear theories. Although past studies considered the effect of the Type I migration of protoplanetary embryos, in most cases they used a conventional formula based on static torques in isothermal disks, and employed a reduction factor to account for uncertainty in the mechanism details. However, in addition to static torques, a migrating planet experiences dynamic torques that are proportional to the migration rate. These dynamic torques can impact on planet migration and predicted planetary populations. In this study, we derived a new torque formula for Type I migration by taking into account dynamic corrections. This formula was used to perform population synthesis simulations with and without the effect of dynamic torques. In many cases, inward migration was slowed significantly by the dynamic effects. For the static torque case, gas giant formation was effectively suppressed by Type I migration; however, when dynamic effects were considered, a substantial fraction of cores survived and grew into gas giants.

1. Introduction

The population of extrasolar planets (Baruteau and Papaloizou, 2013), and perhaps even the solar system (Walsh et al., 2011), provide strong evidence that migration has played a role in shaping planetary systems. Low-mass planets (i.e., those with masses up to that of Neptune) migrate through the excitation of linear density waves in the disk, and through a contribution from the corotation region (i.e., Type I migration). Early analytical work (Tanaka et al., 2002) focused on isothermal disks, in which the temperature was prescribed and fixed. These studies found that migration was always directed inward for reasonable disk parameters, and that migration time scales were much shorter than the disk life time; therefore, according to migration theory, all the planets should end up very close to the central star.

While Type I migration has always been linked to linear interactions with the disk, Paardekooper and Papaloizou (2009) showed that corotation torque (or horseshoe drag) in isothermal disks show nonlinear behavior and can be much larger than previous linear estimates, which works against fast inward migration. However, the corotation tends to be prone to saturation and fail to prevent rapid inward migration for most of the cases, since in the absence of a diffusive process, the corotation region is a closed system; therefore, it can only provide a finite amount of angular momentum to a planet.

Several well-established theoretical models of planet formation based on the core accretion scenario adopted a population synthesis approach (e.g., Ida and Lin, 2004, 2008; Mordasini et al., 2009a,b; Ida et al., 2013). Ida and Lin (2004) focused on the influence of Type I migration on planetary formation processes and found that when the effects of Type I migration are taken into account, planetary cores have a tendency to migrate into their host stars before they acquire adequate mass to initiate efficient gas accretion. In order to preserve a sufficient fraction of gas giants around solar-type stars, they introduced a Type I migration reduction factor, where factor magnitudes of smaller than unity work to lengthen the Type I migration timescale relative to those deduced from linear theories. With a range of small factors (~ 0.01), it was possible to produce a planetary $M_p - a$ distribution that was

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qualitatively consistent with observations from a radial velocity survey. While several suppression mechanisms for Type I migration under various circumstances have been suggested (e.g., Paardekooper et al., 2011), the origin of the extremely small reduction factor values remains unknown.

Recently, it was proposed that dynamic corotation torque can also play a role for low-mass planets, especially where static corotation torques saturate. Paardeokooper (2014) presented an analysis of the torques on migrating, low-mass planets in locally isothermal disks. They found that planets experience dynamic torques whenever there is a radial gradient in vortensity in addition to static torques, which do not depend on the migration rate. These dynamic torques are proportional to the migration rate and can have either a positive or a negative feedback on migration, depending on whether the planet is migrating with or against the static corotation torque. Moreover, they showed that in disks a few times more massive than the minimum mass solar nebula (MMSN), the effects of dynamic torques are significant to reduce inward migration.

In this study, we deduced a torque formula for Type I migration by taking into account dynamic corrections. Using this formula, we performed population synthesis simulations with and without the dynamic corrections in order to evaluate the migration velocity quantitatively. We found that the effective torques with dynamic correction were much smaller than the simple static torques when applied to disks of the MMSN model. We used dynamic torques based on the theory of Paardeokooper (2014) and estimated actual Type I migration, and then simulated various sets of planetary systems based on the observed range of disk properties. Finally, we compared the simulated results with observational data.

2. Dynamic correction of Type I migration formula

According to Paardekooper et al. (2011; hereafter Pa11) and Coleman and Nelson (2015; hereafter CN14), new static torque formula for Type I migration can be derived (see Appendix). When developing the dynamic correction formula, we considered the work of Paardeokooper (2014), who showed that $\Gamma_{\rm dynamic}$, the term proportional to $dr_{\rm p}/dt$, must be included in the torque formula, or in other words:

$$\Gamma = \Gamma_{\rm static} + \Gamma_{\rm dynamic} \quad (1)$$

which is given by (Pa14 Eq. 18):

$$\Gamma_{\rm dynamic} = 2\pi(1 - w_{\rm c}/w(r_{\rm p}))\Sigma r_p^2 x_{\rm s} \Omega v_{\rm p} \quad (2)$$

where Σ is the surface density of the disk, r_p is the semimajor axis of the protoplanet, x_s is the thickness of the horseshoe region, $\Omega = (GM_*/r_{\rm p}^3)^{1/2}$ (G is the gravitational constant, and M_* is the stellar mass), and $v_{\rm p} = dr_{\rm p}/dt$ is the radial velocity of the protoplanet. Here, $1-w_{\rm c}/w(r_{\rm p})$ was calculated by (Pa14 Eq. 28, modified by TE):

$$1 - w_{\rm c}/w(r_{\rm p}) = (3/2 + p)\min\left(1, \frac{x_{\rm s}^2}{6r_{\rm p}\nu} v_{\rm p}\right) \quad (3)$$

where $p = d\ln\Sigma/d\ln r$ and ν is the viscosity of the disk. Assuming a circular orbit of the protoplanet, $v_{\rm p}$ can be calculated by the equation:

$$\tau_{\rm lib}\frac{dv_{\rm p}}{dt} = -v_{\rm p} + \frac{2q_{\rm d}}{\pi q r_{\rm p}^3 \Omega \Sigma}\left(\Gamma_{\rm static} + \Gamma_{\rm dynamic}\right) \quad (4)$$

where $\tau_{\rm lib} = 4\pi r_{\rm p}/(3\Omega x_{\rm s})$ is the liberation timescale of gas in the disk, $q = M_{\rm p}/M_*$, and $q_{\rm d} = \pi r_{\rm p}^2\Sigma/M_*$. For the case of slow migration (i.e., $\tau_{\rm lib} \ll 4\pi r_{\rm p}/(3\Omega x_{\rm s}) \ll r_{\rm p}/v_{\rm p}$), we were able to assume a steady state for Eq. 4 to determine $v_{\rm p}$, or in other words:

$$-v_{\rm p} + \frac{2q_{\rm d}}{\pi q r_{\rm p}^3 \Omega \Sigma}\left(\Gamma_{\rm static} + \Gamma_{\rm dynamic}\right) = 0 \quad (5)$$

2.1. Inviscid case $\frac{x_s^2}{6r_p\nu}v_p > 1$

We derived $v_{\rm p}$ by substituting Eqs. 2 and 3 into Eq. 4 as:

$$\Gamma_{\rm inviscid} = \frac{1}{1 - (3/2 + p)m_{\rm c}}\Gamma_{\rm static} \quad (6)$$

where, $m_{\rm c}$ is given by (Pa14 Eq. 20):

$$m_{\rm c} = 4q_{\rm d}\bar{x}_{\rm s}/q \quad (7)$$

where $\bar{x}_{\rm s} = x_{\rm s}/r_{\rm p}$

2.2. Viscid case $\frac{x_s^2}{6r_p\nu}v_p < 1$

We derived a quadratic equation of $v_{\rm p}$ by substituting Eqs. 2 and 3 into Eq. 4 as:

$$Av_{\rm p}^2 + Bv_{\rm p} + C = 0 \quad (8)$$

where

$$A = \frac{2q_{\rm d}\bar{x}_{\rm s}^3 r_{\rm p}}{3q\nu}\left(\frac{3}{2} + p\right) = m_{\rm c}\left(\frac{3}{2} + p\right)\frac{\tau_\nu}{6r_{\rm p}} \quad (9)$$

$$B = -1 \quad (10)$$

$$C = \frac{2q_{\rm d}q}{\pi h^2} r_{\rm p}\Omega\gamma_{static} = \frac{r_{\rm p}}{\tau_{\rm mig}}\gamma_{\rm static} \quad (11)$$

where h is the scale height of the disk, $\gamma_{\rm static} = \Gamma_{\rm static}/\Gamma_0$ ($\Gamma_0 = (q/h)^2\Sigma r^4\Omega^2$), and τ_ν and $\tau_{\rm mig}$ are the timescales of diffusion and migration, respectively, as given by (Pa14 Eqs. 10 and 23):

$$\tau_\nu = \frac{r_{\rm p}^2\bar{x}_{\rm s}^2}{\nu} \quad (12)$$

$$\tau_{\rm mig} = \frac{\pi h^2}{2q_{\rm d}q\Omega} \quad (13)$$

The quadratic formula gives the total torque after dynamic correction:

$$\Gamma_{\rm viscid} = \Theta(k)\Gamma_{\rm static} \quad (14)$$

where the function $\Theta(k)$ is defined by (Pa14 Eq. 30):

$$\Theta(k) = \frac{1 - \sqrt{1 - 2k}}{k} \quad (15)$$

where k is the coefficients given by (Pa14 Eqs. 31–32):

$$k = \frac{8}{3\pi}\left(\frac{3}{2} + p\right)\frac{\gamma_{\rm static}q_{\rm d}^2\bar{x}_{\rm s}^3}{h^2}\frac{r_{\rm p}\Omega}{\nu} = \left(\frac{3}{2} + p\right)\frac{m_{\rm c}\tau_\nu\gamma_{\rm static}}{6\tau_{\rm mig}} \quad (16)$$

The function $\Theta(k)$ takes a critical value of 2 at $k = 1/2$, but for $k > 1/2$ it does not take any value, since the inside of the square root of $\Theta(k)$ becomes negative. This suggests that runaway migration takes place for the case $k > 1/2$. Paardeokooper (2014) suggested

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