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A new analytical scaling for turbulent wind-bent plumes: Comparison of scaling laws with analog experiments and a new database of eruptive conditions for predicting the height of volcanic plumes

Thomas J. Aubry[a,*], A. Mark Jellinek[a], Guillaume Carazzo[b], Rose Gallo[a], Kim Hatcher[a], Jack Dunning[a,c]

[a] Department of Earth, Ocean, and Atmospheric Sciences, University of British Columbia, Vancouver, BC, Canada
[b] Observatoire Volcanologique et Sismologique de Martinique, Institut de Physique du Globe de Paris, Sorbonne Paris Cité, CNRS, Morne des Cadets, Fonds St Denis F-97250, Martinique, France
[c] Department of Earth Science and Engineering, Imperial College London, Exhibition Road, London SW7 2AZ,United Kingdom

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ABSTRACT

Various scaling relationships relate the height of volcanic plumes to eruptive source conditions, atmospheric density stratification, turbulent entrainment, and wind stresses. However, observational, analog, and numerical studies used to test these scalings capture only a narrow range of natural eruptive conditions. In particular, existing analytical scalings are not appropriate for the wind stress conditions typical of the majority of volcanic eruptions. Accordingly, we develop a new analytical scaling for the height of buoyant plumes rising in density-stratified uniform crossflows. We compare this scaling to existing analytical scalings (Morton et al. 1956; Hewett et al. 1971) as well as "functional" scalings (i.e., parameterizations expressed as a function of the Morton et al. (1956) scaling and a regime parameter related to wind stress, Degruyter and Bonadonna (2012), Woodhouse et al. (2013), Carazzo et al. (2014)) using the extensive experimental dataset from Carazzo et al. (2014) along with natural events from a new database including 94 eruptive phases. Our proposed scaling best predicts the height of experimental plumes, which enables us to constrain the ratio of the wind to radial entrainment coefficients. For natural eruptions, the Woodhouse et al. (2013) and Carazzo et al. (2014) scalings, which account explicitly for wind gradient, best predicts plume heights. We show that accounting for atmospheric stratification and wind improves empirical relationships between mass eruption rates and plume heights. For tested scalings, analyze of residual heights for natural eruption supports the hypothesis that volcanic plumes rise higher in a wetter atmosphere. Finally, for analog plumes rising under moderate to high wind stresses, we show that plume shapes evolve over the plume height, violating the self-similarity assumption on which all scalings and integral model results rely. We discuss consequences for relaxing the self-similarity hypothesis as well as potential improvements for standard integral plume models, in turn.

1. Introduction

1.1. Overview: volcanic plume height and the problem of wind stresses

Analytical scaling relationships provide important insight into the dynamics of volcanic plumes by relating the plume heights to (i) eruption source conditions, (ii) atmospheric conditions and (iii) the rate of turbulent entrainment of air into rising plumes. Scalings enable a better understanding of the main controls on plume height which, in turn, enables rigorous assessments of societal impacts of explosive eruptions related to, e.g., falling ash (e.g. Carey and Sparks, 1986), climate change on timescales of weeks to centuries (e.g. Robock, 2000; Timmreck, 2012), and the delivery of nutrients to oceans, such as iron (e.g. Langmann et al., 2010; Browning et al., 2015). Furthermore, when tested against analog laboratory experiments, analytical scalings enable the calibration of turbulent entrainment rates (e.g. Morton et al., 1956; Hoult and Weil, 1972; Fischer et al., 1979) which must be specified in more sophisticated plume models (e.g. Degruyter and Bonadonna, 2012; Woodhouse et al., 2013).

A number of recent eruptions strongly impacted by winds (e.g. Eyjafjallajökull in 2010, Cordón Caulle in 2011, Calbuco in 2015) have

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* Corresponding author at: Department of Earth, Ocean and Atmospheric Sciences, University of British Columbia, UBC Vancouver Campus, 2020 - 2207 Main Mall, V6T1Z4 Vancouver, BC,Canada.
E-mail address: taubry@eoas.ubc.ca (T.J. Aubry).

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reanimated debates over how best to parameterize wind effects on turbulent entrainment in buoyant plume models, from which scalings are derived (e.g., Degruyter and Bonadonna, 2012; Woodhouse et al., 2013; Mastin, 2014; Bonadonna et al., 2015c; Woodhouse et al., 2015; Girault et al., 2016; Folch et al., 2016). Accordingly, a practical aim is to provide a self-consistent intercomparison of scalings relating the height of volcanic plumes to wind stresses through their effect on entrainment, as determined from both laboratory experiments and observations of natural eruptions.

1.2. *Plume rising in quiescent conditions: the Morton et al. (1956) scaling*

Most scalings for the rise of volcanic plumes follow the classical approach of Morton et al. (1956). For a fully developed turbulent and incompressible plume with steady source conditions, Morton et al. (1956) develop a similarity theory by making the following assumptions:

1. The turbulent entrainment of ambient air into the plume is driven by lateral pressure gradients related to velocity differences between the plume and the atmosphere. Consequently, the inflow velocity u_ϵ of ambient fluid into the plume is assumed to be proportional to the centerline velocity u of the plume at a height z such that

   $$u_\epsilon = \alpha u \quad , \tag{1}$$

   where α is the radial entrainment coefficient.
2. The profiles of velocity and density across the plume are of similar form at all heights along the plume centerline (e.g., "top-hat" or Gaussian functions).

For a dry, quiescent and linearly stratified atmosphere, and under the Boussinesq assumption, a plume rising from a point source of buoyancy flux with top-hat velocity and density profiles will ascend to a maximum height (Fig. 1, left) given by:

$$H^{MTT} = 2.8 \times \left(2^{-\frac{5}{8}}\pi^{-\frac{1}{4}}\alpha^{-\frac{1}{2}}F_0^{\frac{1}{4}}N^{-\frac{3}{4}}\right) \quad , \tag{2}$$

where the prefactor 2.8 is the non-dimensional maximum plume height, F_0 is the source buoyancy flux of the plume, $N = \sqrt{-\frac{g}{\rho_{a0}}\frac{\mathrm{d}\rho_a}{\mathrm{d}z}}$ is the atmospheric Brunt-Väisälä frequency with g the Earth's gravity acceleration, ρ_a the ambient density, z the altitude, and 0 subscript denoting properties taken at source altitude (cf.Table 1 for a summary of symbols used). Eq. (2) is, thus, an analytical expression that explicitly relates the height of a plume to its source condition, the environmental stratification and the radial entrainment rate.

Explosive volcanic plumes erupt as negatively buoyant jets with dense pyroclasts in the flow. The initial momentum flux drives entrainment of atmosphere, which is heated by hot pyroclasts, in turn reducing the mean density of the jet. If the efficiency of turbulent entrainment and heating from pyroclasts is sufficiently high, the buoyancy of the column reverses before the initial momentum flux is exhausted and the mixture rises by natural convection. The height of the momentum-dominated region is generally small compared to the total rise height, so that Eq. (2) is commonly applied to study the rise of volcanic plumes, where the buoyancy flux is specified in terms of an enthalpy flux (Wilson et al., 1978; Woods, 1995).

The radial entrainment coefficient α is historically calibrated using small-scale laboratory experiments (e.g. Morton et al., 1956; Fischer et al., 1979; Chen and Rodi, 1980; Kaminski et al., 2005), large-scale experiments (e.g. Dellino et al., 2014), or numerical simulations (e.g. Suzuki and Koyaguchi, 2010, 2015). Laboratory experiments show that for radial top-hat-shaped profiles, α lies between 0.065 and 0.07 in pure jets (driven by a source of momentum only,

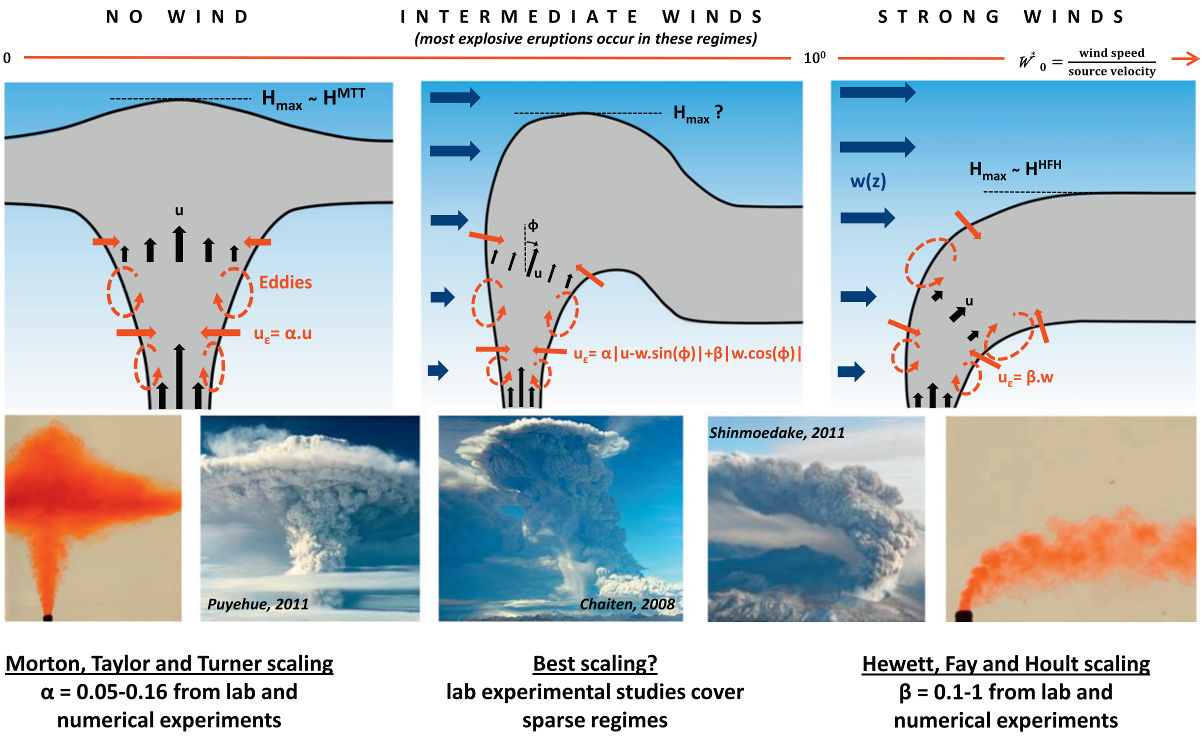

Fig. 1. Entrainment hypothesis and scalings for plume height across different wind regimes: quiescent atmosphere (left), intermediate winds (center), and high winds (right). The different notations are defined in the text and in Table 1. Left (respectively right) picture shows laboratory experiment performed under $\mathcal{W}^*_0 \simeq 10^{-3}$ (respectively $\mathcal{W}^*_0 \simeq 10^0$). Pictures in the middle show eruptions occurring under $\mathcal{W}^*_0$ ranging from 10^{-3} to 10^{-1}.

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