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The influence of pore geometry and orientation on the strength and stiffness of porous rock

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A B S T R A C T

The geometry of voids in porous rock falls between two end-members: very low aspect ratio (the ratio of the minor to the major axis) microcracks and perfectly spherical pores with an aspect ratio of unity. Although the effect of these end-member geometries on the mechanical behaviour of porous rock has received considerable attention, our understanding of the influence of voids with an intermediate aspect ratio is much less robust. Here we perform two-dimensional numerical simulations (Rock Failure Process Analysis, $RFPA_{2D}$) to better understand the influence of pore aspect ratio (from 0.2 to 1.0) and the angle between the pore major axis and the applied stress (from 0 to 90°) on the mechanical behaviour of porous rock under uniaxial compression. Our numerical simulations show that, for a fixed aspect ratio (0.5) the uniaxial compressive strength and Young's modulus of porous rock can be reduced by a factor of ~2.4 and ~1.3, respectively, as the angle between the major axis of the elliptical pores and the applied stress is rotated from 0 to 90°. The influence of pore aspect ratio on strength and Young's modulus depends on the pore angle. At low angles (~0–10°) an increase in aspect ratio reduces the strength and Young's modulus. At higher angles (~40–90°), however, strength and Young's modulus increase as aspect ratio is increased. At intermediate angles (~20–30°), strength and Young's modulus first increase and then decrease as pore aspect ratio approaches unity. These simulations also highlight that the influence of pore angle on compressive strength and Young's modulus decreases as the pore aspect ratio approaches unity. We find that the analytical solution for the stress concentration around a single elliptical pore, and its contribution to elasticity, are in excellent qualitative agreement with our numerical simulations. The results of our numerical modelling are also in agreement with recent experimental data for porous basalt, but fail to capture the strength anisotropy observed in experiments on sandstone. We conclude that the alignment of grains or platy minerals such as clays exerts a greater influence on strength anisotropy in porous sandstones than pore geometry. Finally, we show that the strength anisotropy that arises as a result of preferentially aligned elliptical pores is of a similar magnitude to that generated by bedding in porous sandstones and foliation in low-porosity metamorphic rocks. The modelling presented herein shows that porous rocks containing elliptical pores can display a strength and stiffness anisotropy, with implications for the preservation and destruction of porosity and permeability, as well as the distribution of stress and strain within the Earth's crust.

1. Introduction

Most rocks contain porosity in the form of pores, microcracks, or a combination of the two. Porosity is known to exert a first-order control on the physical properties of rocks. For example, with increasing porosity, strength (e.g., Al-Harthi et al., 1999; Chang et al., 2006; Zhu et al., 2011; Baud et al., 2014; Schaefer et al., 2015) and Young's modulus (e.g., Chang et al., 2006) decrease and permeability increases (e.g., Bourbié and Zinszner, 1985; Farquharson et al., 2015; Wadsworth et al., 2016). These studies have shown that porosity alone (i.e. the scalar quantity) does not control the mechanical and hydraulical behaviour of rocks, highlighting an important role for the geometry of the void space (e.g.,

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Chang et al., 2006; Farquharson et al., 2015). The aspect ratio (the ratio of the minor to major semi axis) of an elliptical void within a rock will fall between two end-members: microcracks that have a very low aspect ratio ($10^{-3} - 10^{-5}$, Simmons and Richter, 1976) and perfectly spherical pores with an aspect ratio of unity. Indeed, recent advances in X-ray micro-computed tomography (μCT) have shown that porous rocks can contain a wide variety of pore shapes (e.g., Ji et al., 2012; Rozenbaum and Rolland du Roscat, 2014; Ji et al., 2015; Schmitt et al., 2016; Luquot et al., 2016; Arzilli et al., 2016; Bubeck et al., 2017; Zambrano et al., 2017).

Although two-dimensional micromechanical and numerical models exist to help understand the influence of uniformly aligned microcracks (e.g., Ashby and Sammis, 1990) and circular pores (e.g., Sammis and Ashby, 1986; Heap et al., 2014) on the mechanical behaviour of porous materials, including rocks, much less is known as to the influence of voids with an intermediate aspect ratio. A recent experimental study has shown, using uniaxial compressive strength tests, that basalt samples containing elliptical pores (aspect ratio ≈ 0.5) oriented with their major axis perpendicular to the loading direction were measurably weaker than those prepared to contain pores with their major axis parallel to loading (Bubeck et al., 2017). Although this study offers insight into the influence of non-spherical pores on mechanical behaviour, it remains challenging to isolate the influence of a specific parameter (e.g., pore aspect ratio, pore orientation with regards to the loading direction, and porosity) using natural samples, a consequence of their inherent variability. To circumvent natural variability, we use here a numerical modelling approach to isolate the role of select pore geometrical parameters (pore aspect ratio and pore orientation) on the compressive strength and Young's modulus of porous materials. We report on the results of numerical simulations, using the two-dimensional Rock Failure Process Analysis code (RFPA$_{2D}$; Tang, 1997), in which we uniaxially deform rectangular samples populated with elliptical pores. Samples were built to contain different porosities (from 0.02 to 0.2), pore aspect ratios (from 0.2 to 1.0), and angles between the pore major axis and the loading direction (from 0 to 90°). Finally, the results of the RFPA$_{2D}$ modelling are compared with two-dimensional analytical solutions for the stress concentration around a single elliptical pore (from Jaeger et al., 2009) the contribution of the elliptical pore to the Young's modulus (from Kachanov et al., 1994). We also compare our modelled results with new and previously published experimental data (basalt, sandstone, and limestone), and compare the strength anisotropy generated by the preferential alignment of elliptical pores, bedding in porous sandstones, and foliation in low-porosity metamorphic rocks.

2. Description of numerical simulations

The two-dimensional Rock Failure Process Analysis code (RFPA$_{2D}$) is a numerical model based on elastic damage mechanics (Tang, 1997). We used the model to uniaxially deform 400 × 200 pixel rectangular bitmap images that contained elliptical pores with a major axis of 20 pixels in length (Fig. 1). Considering a resolution of 0.1 mm/pixel, the images are analogous to rectangular samples 40 mm in length and 20 mm in width containing elliptical pores with a major axis of 2 mm. These samples were generated using a MATLAB script. To generate a bitmap image, we first selected a fixed pore aspect ratio (the ratio of the minor to major semi axis) and a fixed pore angle, β, (the angle measured from the vertical long axis of the rectangle and the major axis of the elliptical pore in a clockwise manner). We then iteratively added pores to the image in random locations until the target porosity was met. Pores were not allowed to overlap, although they could intersect the boundary of the image (Fig. 1). Each 0.1 mm square element was assigned a Young's modulus (E) and a value of compressive (σ_{cr}) and tensile (σ_{tr}) strength. The pores were considered to have zero strength and Young's modulus, whilst the elements comprising the matrix were assigned values of strength and Young's modulus using a Weibull probability density function (Weibull, 1951; Wong et al., 2006):

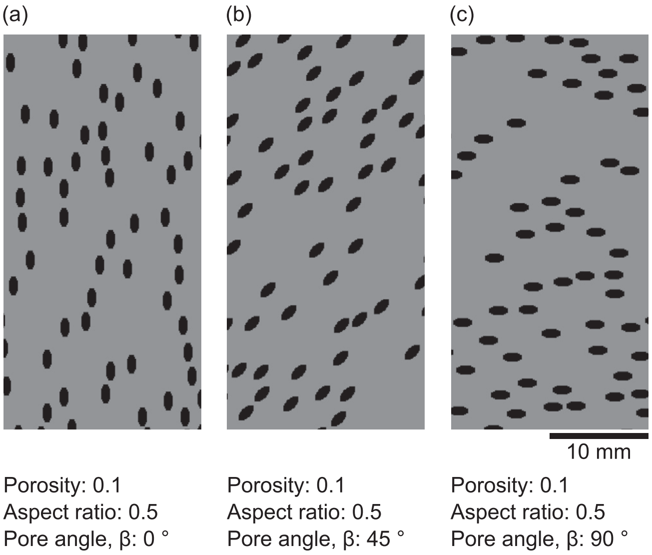

Fig. 1. Examples of randomly generated numerical samples containing a porosity of 0.1: elliptical pores with an aspect ratio of 0.5. Samples are rectangular bitmap images (400 × 200 pixels; 40 × 20 mm) containing elliptical pores with a major axis of 20 pixels in length (i.e. 2 mm). The angles between the vertical long axis of the rectangular sample (i.e. the loading direction) and the major axis of the elliptical pore, β, are (a) 0°, (b) 45°, and (c) 90°.

$$x(u) = \frac{m}{u_0}\left(\frac{u}{u_0}\right)^{m-1} exp\left[-\left(\frac{u}{u_0}\right)^{m}\right] \tag{1}$$

where $x(u)$ is $\sigma_{cr}(u)$, $\sigma_{tr}(u)$, or $E(u)$, and u and u_0 are respectively the scale parameter of an individual element and the scale parameter of the average element (given in Table 1), respectively. We chose a matrix homogeneity factor m—the Weibull shape parameter—of 3 for all of our simulations. High values of m will yield more homogeneous samples (the property of a particular element will be closer to the chosen mean), and vice-versa (see Xu et al., 2012). Examples of the distribution (given by Equation (1)) of Young's modulus (E) and uniaxial compressive strength (σ_{cr}) are provided in Fig. 2 (for a sample containing 80,000 elements, $m = 3$, and the matrix element properties given in Table 1). Our matrix element properties (Table 1) and homogeneity factor ($m = 3$) are the same as those used in the recent publications of Heap et al. (2014; 2015a; 2016). Although, for example, the mean uniaxial compressive

Table 1
The mean physical and mechanical properties of the matrix elements (i.e. the 0.1 mm squares that form the sample) used in the Rock Failure Process Analysis code (RFPA$_{2D}$) numerical modelling. The value assigned to each element was determined using the Weibull probability function (Equation (1); see Fig. 2). The matrix element properties are the same as those used in the recent publications of Heap et al. (2014; 2015a; 2016).

Homogeneity index	3
Mean uniaxial compressive strength (MPa)	2300
Mean Young's modulus (GPa)	100
Poisson's ratio	0.25
Ratio of compressive to tensile strength	10
Frictional angle (°)	30