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A new model for the computation of the formation factor of core rocks

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ABSTRACT

Among all the rock parameters measured by modern well logging tools, the formation factor is essential because it can be used to calculate the volume of oil- and/or gas in wellsite. A new mathematical model to calculate the formation factor is analytically derived from first principles. Given the electrical properties of both rock and brine (resistivities) and tortuosity (a key parameter of the model), it is possible to calculate the dependence of the formation factor with porosity with good accuracy. When the cementation exponent ceases to remain constant with porosity; the new model is capable of capturing both: the non-linear behavior (for small porosity values) and the typical linear one in log-log plots for the formation factor vs. porosity. Comparisons with experimental data from four different conventional core rock lithologies: sands, sandstone, limestone and volcanic are shown, for all of them a good agreement is observed. This new model is robust, simple and of easy implementation for practical applications. In some cases, it could substitute Archie's law replacing its empirical nature.

1. Introduction

Oil and natural gas are considered to be the most important sources of energy in the world. In petroleum industry, well logs play a fundamental role in exploration and reservoir characterization e.g. (Egbai and Aigbogun, 2012). The primary objectives of logging in an exploration site are to locate and to quantify the amount of hydrocarbons in the vicinity of a borehole. There are many different types of well logs including gamma ray, caliper, density, neutron, sonic and electrical resistivity logs, among others e.g. (Yan, 2002). Possibly, the most important and widely studied of these is the electrical resistivity log, which is used routinely to calculate the porosity and saturation of reservoir rocks both quantities related to the interpretation and analysis of the reservoir content. Strictly speaking, the electrical resistivity log is a function of several physical parameters and lithological attributes, including electric resistivities of formation and pore water; temperature, viscosity, and degree of saturation of pore water; type and amount of clays; mechanism of charge fixation at the fluid - solid interface (represented by specific surface area and electric surface conductance); intricate geometry of pores and pore channels (i.e. tortuosity among others parameters); the ratio of the volume of voids to total volume (represented by the effective porosity); formations ability to transmit pore water (represented by permeability); cation exchange capacity; and size, shape, type (mineralogy), packing, sorting, and distribution of grains.

Typically, the study of electrical resistivity of such reservoir formations is done by saturating the rocks core with a brine solution (conducting liquid) in order to measure the electrical resistivity of this bulk system, R_0 (Sheriff, 1974). Since the resistivity of the saturating liquid is known, R_l, it is possible to calculate the formation factor defined as:

$$F = \frac{R_0}{R_l}. \tag{1}$$

Extensive experimental work has shown that the bulk resistivity of a rock, R_0, depends on its porosity, pore fluid resistivity and saturation (Waxman and Smits, 1968; Waxman, 1974; Clavier et al., 1977; Givens, 1987). Consequently, from a theoretical point of view, most of the efforts have been focused on developing mathematical relationships between water resistivity, porosity and water

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List of symbols

R_0	Electrical resistivity of the bulk system
R	electrical resistivity
r	symbol for the rock phase
l	symbol for the liquid phase
F	formation factor
Φ	porosity
m	cementation exponent
G	tortuosity
φ	dimensionless electrical current density
ξ	dimensionless radial coordinate
b	pore-rock system radius
r^*	dimensional radial coordinate
J	dimensional current density at any radius
J_a	dimensional current density at the pore radius
δ	skin depth parameter
ε	ratio of the skin depth parameter δ and pore size.
μ	magnetic permeability of the medium
ω	frequency of the electrical signal
β	fitting parameter
c	aspect ratio parameter
m_A	cementation exponent based on Archie's law
m_t	average cementation exponent
ΔT_c	characteristic temperature drop.
θ	dimensionless temperature
ϕ	temperature coefficient for resistivity

saturation. In a seminal work published by Archie (1942), an empirical relationship between the formation factor and porosity (Φ) for different lithologies was formulated as:

$$F = \Phi^{-m} \tag{2}$$

where the constant m is often known as the cementation exponent. The cementation exponent provides implicit information about the pore structure. Equation (2) is commonly referred to as Archie's law and it is still the most widely used expression for resistivity log interpretation. Practical applications of Archie's law in a particular area requires a knowledge of Φ and m from the laboratory measurements or studies of a well logging data in such area. Despite its prevalence and simplicity, Equation (2) is, in fact, a coarse and an oversimplified model. Its limitations are evident when contrasted with various experimental data.

The study of the formation factor constitutes in itself a classic subject of petrophysics research (Archie, 1942; Pérez-Rosales, 1982; Saner et al., 1996), since it provides a useful and convenient evaluation of the nature of the pore structure of reservoir rocks. Archie's law, according to Equation (2) states that F is a function of a power of Φ; when plotted in a log-log scale, the relation appears as a straight line whose slope is equal to the power $-m$ (the cementation exponent). It has been observed and widely reported that, when the range of porosities is extended (specially for small values), the data does not follow a power-law trend with a constant exponent, i.e., the lines are curved in a log-log scale, (Givens, 1987; Worthington, 1993; Padhy et al., 2006); this non-Archie behavior has been qualitatively explained by theoretical models, based on simple mixing laws (Petricola and Watfa, 1995; Fleury, 2002) or effective medium approximation (EMA) (Sen, 1997). Therefore, to obtain better predictions for the entire range of porosities and for different rock lithologies, improvements to Archie's law or new models are needed. To our knowledge, just a few attempts to model the non-constant power relation between F and Φ have appeared in the open literature (Givens, 1987; Glover et al., 2000; Bernabé et al., 2011; Nguyen, 2014).

Recently, due to the development of new software and tools in the fields of image acquisition/processing, it is possible to perform a more detailed electrical characterization of core rocks (Knackstedt et al., 2007; Elmer, 2009; Schlumberger, 2014); unfortunately, they are expensive.

In the present article we develop an expression to evaluate the formation factor, defined by Equation (1), as a function of the rock porosity and its physical properties. Our model is based on a more general study recently developed by Chávez et al. (2014), which accounted for the coupled thermo-electrical effects of a composite material to predict the electrical resistivity. Considering the range of physical properties of rocks, Chávez et al. model can be significantly simplified and an analytical expression for the formation factor is, in fact, obtained. Some comparisons between our approach and another theoretical model for the formation factor of sandstone lithologies are presented; differences and common points among both theories are pointed out. To fully validate the present model, we also conducted comparisons with experimental data for different conventional rock cores lithologies and porosities, leading to a good agreement.

The paper is organized as follows. In Section 2, we describe the mathematical formulation and the assumptions under which our problem will be set to compute the formation factor. This is followed in Section 3 with comparisons between numerical results and experimental data as well as some discussions are presented. Finally, some concluding remarks are drawn in Section 4.

2. Mathematical model

In our recent theoretical work (Chávez et al., 2014), a nonlinear conjugate thermo-electric model was developed to study the effect of frequency, resistivities and thermal conductivities on the current density and temperature profiles for a two-phase composite medium. Implementing a numerical solution in cylindrical coordinates, current density and temperature profiles were obtained for each phase. Additionally, based on Equation (1), we established an expression to evaluate the formation factor in terms of the dimensionless current densities for each phase as:

$$F = \frac{\frac{G}{\Phi}\int_0^{\sqrt{\Phi/G}} \varphi_l \xi d\xi + \frac{G}{G-\Phi}\frac{R_r}{R_l}\int_{\sqrt{\Phi/G}}^{1} \varphi_r \xi d\xi}{2\left(\int_0^{\sqrt{\Phi/G}} \varphi_l \xi d\xi + \int_{\sqrt{\Phi/G}}^{1} \varphi_r \xi d\xi\right)}. \tag{3}$$

where the symbols l and r stand for liquid and rock phases respectively, R is the electrical resistivity, G is the tortuosity, φ is the dimensionless electrical current density, ξ is the dimensionless radial coordinate, these last two variables are respectively defined as:

$$\xi = \frac{r^*}{b} \tag{4}$$