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Stable and efficient Q-compensated least-squares migration with compressive sensing, sparsity-promoting, and preconditioning

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ABSTRACT

The anelastic effects of subsurface media decrease the amplitude and distort the phase of propagating wave. These effects, also referred to as the earth's Q filtering effects, diminish seismic resolution. Ignoring anelastic effects during seismic imaging process generates an image with reduced amplitude and incorrect position of reflectors, especially for highly absorptive media. The numerical instability and the expensive computational cost are major concerns when compensating for anelastic effects during migration. We propose a stable and efficient Q-compensated imaging methodology with compressive sensing, sparsity-promoting, and preconditioning. The stability is achieved by using the Born operator for forward modeling and the adjoint operator for back propagating the residual wavefields. Constructing the attenuation-compensated operators by reversing the sign of attenuation operator is avoided. The method proposed is always stable. To reduce the computational cost that is proportional to the number of wave-equation to be solved (thereby the number of frequencies, source experiments, and iterations), we first subsample over both frequencies and source experiments. We mitigate the artifacts caused by the dimensionality reduction via promoting sparsity of the imaging solutions. We further employ depth- and Q-preconditioning operators to accelerate the convergence rate of iterative migration. We adopt a relatively simple linearized Bregman method to solve the sparsity-promoting imaging problem. Singular value decomposition analysis of the forward operator reveals that attenuation increases the condition number of migration operator, making the imaging problem more ill-conditioned. The visco-acoustic imaging problem converges slower than the acoustic case. The stronger the attenuation, the slower the convergence rate. The preconditioning strategy evidently decreases the condition number of migration operator, which makes the imaging problem less ill-conditioned and significantly expedites the convergence rate. Numerical experiments verify the stability, benefits and robustness of the method proposed and support our analysis and findings.

1. Introduction

Due to the fact that real earth media are anelastic, the propagation of seismic waves in field media is in many respects significantly different from propagation in perfectly elastic media (Mittet et al., 1995). The anelastic properties of the subsurface media cause dissipation of wave energy, thus decreasing the amplitude and distorting the frequency content of the wavelet propagating inside the earth (Quan and Harris, 1997). Fluids and especially gas trapped in overburden structures cause strong attenuation and dispersion of the seismic wave, as observed over many oil-gas fields (Zhu et al., 2014). A quality factor Q was introduced to quantify the anelastic attenuation and dispersion effects (Futterman, 1962; Kjartansson, 1979; Knopoff, 1964; Kolsky, 1956; Morozov and Ahmadi, 2015), which are now generally referred to as Q filtering effects (Wang, 2006). The term Q is used to represent the energy lost for a wavefield propagating along one wave length and is often defined as $Q = 2\pi E/\Delta E$, where E is the energy of the wavefield and ΔE is the energy lost in a wavelength of propagation.

Ignoring the Q filtering effects during seismic migration/imaging processes, for example, acoustic reverse time migration (RTM) (e.g., Baysal et al., 1983; Chattopadhyay and McMechan, 2008; Levin, 1984)

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and acoustic least-squares migration (LSM) (e.g., Herrmann and Li, 2012; Kazemi and Sacchi, 2015; Nemeth et al., 1999; Wu et al., 2015), would produce an image with reduced amplitude and incorrect position of reflectors, especially those below highly attenuating geologic environments (Zhu et al., 2014). To extract more detailed information from *Q*-filtered seismic data, we must account for the *Q* filtering effects. By taking the anelastic effects into account during migration, the resultant image is expected to have high resolution with true amplitude and correct timing, which are important for both regional geophysics investigation and reservoir exploration. Mittet et al. (1995) showed that even when imaging with a *Q* model deviating by 10% from the true one, images would still be better focused and of a higher quality than when no *Q*-compensation was performed (Wang, 2008). There are methods mitigating the *Q* filtering effects based on the one-dimensional (1D) wave equation (e.g., Chai et al., 2017, 2016a, 2014; Wang, 2006). However, because the anelastic effects on recorded seismic data occur during the wave propagation and are related to wavepaths for 2D/3D surveys, it is more accurate and physically more consistent to address these effects in a way of 2D/3D wave-equation based pre-stack depth migration.

Conventional method to compensate for the *Q* filtering effects is to reverse the sign of attenuation operator. The method is unstable, since it supports waves with exponential growth. The amplitudes of high-frequency components grow rapidly in the wavefield extrapolation, numerical round-off errors tend to be amplified drastically. This causes a huge amount of undesirable noise in the image, even if the input data are noise-free (Wang, 2006, 2008). Fig. 1 illustrates the intrinsic numerical instability associated with inverse *Q*-compensation. The scale of the amplitude compensation by reversing the sign of the attenuation operator grows rapidly (see Fig. 1b), which introduces a large number of artifacts into the inverse *Q* filtered seismogram (see Fig. 1c). The numerical instability is a major concern when compensating for the *Q* filtering effects (Li et al., 2016; Zhu et al., 2014).

Mittet et al. (1995) design one-way compensation extrapolators for up- and down-going waves to address the complete amplitude loss from source to receiver. To stabilize the results, they use a high-cut frequency filter on the initial state for up- and down-going wavefields. Zhang and Wapenaar (2002) suggest limiting the number of extrapolation steps and the maximum angle of migration dip to derive a conditionally stable extrapolation operator. Mittet (2007) compensates for the *Q* filtering effects in deriving the space-frequency domain depth extrapolation operators by starting from the ones without such compensation, and needs to consider the stability problem. To stabilize migration with inverse-*Q* filtering (Wang, 2008), an inverse method implemented by adding a small stabilization factor in the inverse of the operator, is required and implicitly introduces a gain limit. Zhang et al. (2013) develop an effective *Q* method of compensating for absorption and dispersion in migration. Stabilization is achieved by introducing a smooth, maximum-limited gain function that matches the exact amplitude compensation factor when it is less than the user-specified gain limit. Zhu et al. (2014) propose an approach of compensating for attenuation effects in reverse-time migration (*Q*-RTM), in which the attenuation- and dispersion-compensated operators were constructed by reversing the sign of attenuation operator and leaving the sign of dispersion operator unchanged. They design a low-pass filter to stabilize the compensating procedure. The visco-acoustic migration methods described in Sun et al. (2015), Li et al. (2016) also apply low-pass filters to prevent high wavenumber noises from growing exponentially.

Based on a standard linear solid model (Carcione et al., 1988; Zhu et al., 2013), Dutta and Schuster (2014) use the time-domain visco-acoustic wave equation for wavefield extrapolation and use the linearized least-squares inversion method to compensate for the attenuation loss. The method is denoted as *Q*-LSRTM. They need not to regularize the adjoint wave equations during the receiver-side wavefield extrapolation to compensate for the attenuation loss. Their method is stable. The shortcoming of *Q*-LSRTM is that this method is expensive (the computational cost per iteration is more than six times that of standard RTM, and the total computational cost is proportional to the number of least-squares iterations).

With the sparse-recovery and randomized sampling theory from compressive sensing (CS) (Candès et al., 2006b; Donoho, 2006), and the depth- and *Q*-preconditioning strategies, we propose a stable and efficient *Q*-compensated iterative imaging approach in the frequency-domain. The frequency domain is the natural domain for dealing with dispersion and attenuation effects (Song and Williamson, 1995). First, attenuation parameters and dispersive velocity relationships can be incorporated into forward modeling through the use of complex-valued frequency-dependent velocity (Marfurt, 1984; Prieux et al., 2013; Song et al., 1995; Tarantola, 1988). Both phase velocity and attenuation coefficients are allowed to vary with frequency and spatial location. Second, it is flexible for implementing any kind of viscous attenuation mechanism. Besides, for certain geometries, only a few frequency components are required to perform wave-equation based inversion and imaging (Chen et al., 2013; Pratt et al., 1998).

During the imaging process, we use the Born operator for forward modeling and the adjoint operator for back propagating the residual wavefields, thus avoiding the construction of attenuation compensation operator. To reduce the computational cost, which is proportional to the number of partial-differential equation (PDE) to be solved (thereby the number of frequencies, source experiments and iterations), we subsample over both frequencies and source experiments. We suppress the artifacts caused by the dimensionality reduction by promoting sparsity of the imaging solutions. Thus we are using a linearized *Q* inversion method (*Q*-LSRTM) like Dutta and Schuster (2014) combined with sparsity promoting imaging techniques to compensate for the attenuation loss during migration. Since seismic attenuation may significantly slow down the convergence rate of the least-squares iterative inversion process without proper preconditioning (Sun et al., 2016), we employ a depth-preconditioning operator and a *Q*-preconditioning operator to accelerate the convergence rate of iterative migration. Following Herrmann et al. (2015), Yang et al. (2016), we adopt a relatively simple linearized Bregman (LB) method (Lorenz et al., 2014b) to solve the sparsity-promoting visco-acoustic imaging problem. Singular value decomposition (SVD) analysis of the forward operator reveals that attenuation increases the condition number of migration operator, making the imaging problem more ill-conditioned. The stronger the attenuation, the slower the convergence rate. The preconditioning operator evidently reduces the condition number of migration operator making the imaging problem less ill-conditioned, and accelerates the convergence rate of LSM.

The rest of this paper is organized as follows. After notation declarations, we briefly introduce the basic theory of visco-acoustic seismic modeling and imaging. We then describe the proposed approach in detail. Numerical experiments serve to support our analysis and findings. After discussing the limitations and extensions of the method, we summarize our conclusions.

2. Theory

2.1. Notations

Throughout this paper, unless specified otherwise, lowercase boldface symbols (e.g., $\mathbf{x}$) refer to column vectors and uppercase boldface symbols (e.g., $\mathbf{X}$) refer to matrices or operators. The *j*th element of a vector $\mathbf{x}$ is denoted as x_j. $\mathbf{I}$ is the identity matrix. $\mathbf{0}$ is the zero vector. $\log(\cdot)$ is the natural logarithm function. $\log_b(x)$ denotes the logarithm of x to base b. i is the imaginary unit. $\mathrm{diag}(\cdot)$ when