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2D data-space cross-gradient joint inversion of MT, gravity and magnetic data

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ABSTRACT

We have developed a data-space multiple cross-gradient joint inversion algorithm, and validated it through synthetic tests and applied it to magnetotelluric (MT), gravity and magnetic datasets acquired along a 95 km profile in Benxi-Ji'an area of northeastern China. To begin, we discuss a generalized cross-gradient joint inversion for multiple datasets and model parameters sets, and formulate it in data space. The Lagrange multiplier required for the structural coupling in the data-space method is determined using an iterative solver to avoid calculation of the inverse matrix in solving the large system of equations. Next, using model-space and data-space methods, we inverted the synthetic data and field data. Based on our result, the joint inversion in data-space not only delineates geological bodies more clearly than the separate inversion, but also yields nearly equal results with the one in model-space while consuming much less memory.

1. Introduction

In geophysics, accurate prediction of complex subsurface environments requires combining information from multiple geophysical data. Separate inversion methods of each data type are accompanied by the problem of non-uniqueness due to spatial and temporal limitations of observations (Tarantola, 2004), thus one often incorporates some constraints into the inversion procedure to seek reasonable solutions. Regularization, a typical constraint, stabilizes the inversion and creates a model with certain characteristics. Other constraints used in joint inversions include some relationships between different physical properties, concerned with the geophysical data used, such as petrophysical relationships or structural similarity between multiple physical property distributions. The constraints to these relationships can reduce the non-uniqueness inherent in the inversion, and furthermore can avoid inconsistency between models from separate data inversions. The merits of the joint inversion are the ability to reduce the null space for one type of data by the other, and to mutually suppress the influence of different noise sources on individual data (Julia et al., 2000; Moorkamp et al., 2011).

An apparently simple situation is a joint inversion of complementary geophysical measurements that sense the same physical properties. For example, apparent resistivity of DC and MT surveys are both usually modeled in terms of electrical resistivity (Vozoff and Jupp, 1975; Sasaki, 1989). There are also direct coupling approaches involving the use of empirical-observed or analytical-established relationships between different geophysical properties (e.g. petrophysical characteristics relate resistivity and seismic velocity in porous media) (Tillmann and Stöcker, 2000). The direct parameter relationships provide a strong coupling between different datasets and then can lead to good results in joint inversions. But in many cases petrophysical relationships may be largely uncertain and their assumption may not valid for all structures in the region under investigation; therefore the direct coupling approach may result in spurious features (Moorkamp et al., 2011).

One strategy to combine disparate data from unrelated physical phenomena is a joint inversion subject to the conditions of structural similarity. The structure-coupled joint inversion is based on the assumption that the rock structure or boundaries may coincide between the different physical parameter models sensitive to different geophysical methods and thus serve for multiple property coupling and integration (Gallardo and Meju, 2011). In original approaches of structure-coupled joint inversion (Haber and Oldenburg, 1997; Zhang and Morgan, 1996), structural similarity was measured by using model curvatures. These approaches assume that the coincident occurrence of sharp edges or prescribed smoothed edges is the main indicator of structural similarity. However, it can ignore direction-dependent morphological information of boundaries between different properties (Gallardo and Meju, 2011).

Gallardo and Meju (2003) proposed an algorithm based on a new criterion of structural resemblance, so-called cross-gradient function.

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It does not directly deal with the direction vector of the physical property fields that requires normalized gradients and thus can detect differences within large or small gradients without any discontinuities or singularities. Zhdanov et al. (2012) introduced a more generalized approach of joint inversion using Gramian spaces of model parameters and Gramian constraints. The Gramian constraint, when applied to the parameters' gradients, reduces to a constraint similar to cross-gradient. The approaches that minimize cross-gradient have proven to be useful and stable through various applications in multiple joint inverse problems at present (Moorkamp et al., 2011; Gallardo and Meju, 2011; Gallardo et al., 2005, 2012 and references therein).

There exist several optimization algorithms for geophysical inverse problems, including joint inversion, such as the Gauss-Newton (GN) method (Gallardo and Meju, 2004; Gallardo et al., 2012), Quasi-Newton (QN) method (Moorkamp et al., 2011) and conjugate gradient (CG) type method (Moorkamp et al., 2011; Zhdanov et al., 2012). In the separate inversion, the main advantages of GN-type are its stability and robustness (e.g. Loke and Dahlin, 2002). With limited memory optimization schemes such as QN and CG, the full Jacobian matrix and the large and dense coefficient matrix of the system of equations are not necessarily constructed. Probably, these optimization schemes are the most practical choice for large-scale 3D inversion (Commer and Newman, 2008; Avdeev and Avdeeva, 2009; Moorkamp et al., 2011). Meanwhile, GN-type inversions require fewer iterations to converge to the solution than limited memory methods (Loke and Dahlin, 2002; Siripunvaraporn and Egbert, 2007).

In this research, we describe a data-space method based on GN-type joint inversion in Gallardo et al. (2012). With the data-space approach, the part of the parameter space that has no effect on the data is eliminated and the system of normal equations in the model-space is replaced by a system in the data-space (Siripunvaraporn et al., 2005a). In general cases, the number of data parameters is less than that of model parameters by a large factor, and thus the memory required to store the system of equations is much smaller in the data-space method than in the model-space method. For this reason, Gallardo and Meju (2011) indicated that the data-space approach might be applied to structure-coupled joint inversion, while discussing the computational efficiency. Actually, data-space methods have been widely applied to geophysical inversions and other fields (e.g. Parker, 1994; Siripunvaraporn et al., 2005a and references therein).

In this research, we restrict our consideration to the 2D case. The real subsurface structures are three-dimensional and thus 3D inversions often produce more accurate results than 2D inversions. However, when treating regional-scale problems or surveying areas truly close to a 2D structure, field data are collected along a single profile or widely separated profiles. For profiles with dense sites and fine discretization of model, 3D inversion (e.g. 3D MT inversion) can be computationally too expensive; rather 2D inversion is more practical. Thus, we assert that it is still necessary to advance 2D joint inversion.

In this paper, we first describe the model-space formulae for GN-type generalized multiple cross-gradient joint inversion, and then propose the formulae and scheme for data-space implementation. We next discuss the model discretization with topography, the forward modeling, the sensitivity calculation and the algorithm implementation, and compare the memory storage of model-space and data-space methods. Finally, the inversion results from synthetic and real field data examples are compared and discussed.

2. Methodology

2.1. Generalization of cross-gradient joint inversion

2.1.1. Objective function

We start our consideration from the objective function for a separate inversion. The most widely applied approaches to geophysical inversion are regularization schemes based on minimizing a penalty function represented as

$$\varphi = (\mathbf{d}-\mathbf{f}(\mathbf{m}))^{\mathrm{T}}\mathbf{C}_{\mathbf{d}}^{-1}(\mathbf{d}-\mathbf{f}(\mathbf{m})) + \alpha(\mathbf{m}-\mathbf{m}_0)^{\mathrm{T}}\mathbf{C}_{\mathbf{m}}^{-1}(\mathbf{m}-\mathbf{m}_0) \tag{1}$$

(e.g. Constable et al., 1987; Siripunvaraporn et al., 2005a). In this expression $\mathbf{C_d}$ and $\mathbf{C_m}$ are covariances of observed geophysical data $\mathbf{d}$ and a model parameter $\mathbf{m}$, respectively; $\mathbf{m}_0$ is a priori model parameter, and α is a damping parameter. In Eq. (1), the first data misfit term makes the inversion model to satisfy the observed data, and the second regularization term is to overcome non-uniqueness and computational instability of the inversion.

The multiple cross-gradient joint inversion is to search for subsurface images that are geometrically similar and satisfy given datasets within their observation errors (Gallardo, 2007). The objective function of joint inversion is thus composed of data misfits and regularization terms as in Eq. (1) and is constrained by a cross product of model gradients equal to zero (Gallardo and Meju, 2004; Gallardo et al., 2012). The relevant minimization problem is specified by:

$$\begin{aligned}\min: \varphi = &\sum_i (\mathbf{d}_i-\mathbf{f}_i(\mathbf{m}))^{\mathrm{T}}\mathbf{C}_{\mathbf{d}i}^{-1}(\mathbf{d}_i-\mathbf{f}_i(\mathbf{m}))\\ &+\sum_j \alpha_j(\mathbf{m}_j-\mathbf{m}_{0j})^{\mathrm{T}}\mathbf{C}_{\mathbf{m}j}^{-1}(\mathbf{m}_j-\mathbf{m}_{0j})\\ \text{subject to } &\tau(\mathbf{m}_j,\mathbf{m}_l)=0, \forall j \neq l.\end{aligned} \tag{2}$$

In Eq. (2) $\mathbf{f}_i$ represents the forward-modeling operator for the i-th geophysical method and $\mathbf{m}_j$ is the parameter set for the j-th physical model. For generality, we assume that the numbers of i and j are different from each other, and each dataset is sensitive to all the sets of model parameters. For brevity, we omit petrophysical relationships between different sets of model parameters. In the 2D case, model gradients lie on a 2-D plane perpendicular to the strike direction, so the cross-gradient function

$$\tau(\mathbf{m}_j,\mathbf{m}_l) = \nabla\mathbf{m}_j \times \nabla\mathbf{m}_l \tag{3}$$

will only have nonzero components in the strike direction. For convenience, we introduce an integrated model parameter $\mathbf{m} = [\mathbf{m}_1^{\mathrm{T}},\mathbf{m}_2^{\mathrm{T}},\ldots]^{\mathrm{T}}$ and an integrated data parameter $\mathbf{d} = [\mathbf{d}_1^{\mathrm{T}},\mathbf{d}_2^{\mathrm{T}},\ldots]^{\mathrm{T}}$. Then the objective function in Eq. (2) can be rewritten as

$$\begin{aligned}&\min: \varphi = \varphi_{\mathbf{d}} + \varphi_{\mathbf{m}}\\ &\text{subject to } \boldsymbol{\tau} = \mathbf{0},\end{aligned} \tag{4}$$

where

$$\begin{aligned}\varphi_{\mathbf{d}} &= (\mathbf{d}-\mathbf{f}(\mathbf{m}))^{\mathrm{T}}\mathbf{C}_{\mathbf{d}}^{-1}(\mathbf{d}-\mathbf{f}(\mathbf{m})),\\ \varphi_{\mathbf{m}} &= (\mathbf{m}-\mathbf{m}_0)^{\mathrm{T}}\mathbf{C}_{\mathbf{m}}^{-1}(\mathbf{m}-\mathbf{m}_0),\\ \mathbf{f}(\mathbf{m}) &= \left[\mathbf{f}_1^{\mathrm{T}}(\mathbf{m}), \mathbf{f}_2^{\mathrm{T}}(\mathbf{m}), \cdots\right]^{\mathrm{T}}.\end{aligned}$$

The $\mathrm{C}_{\mathbf{d}}$ and $\mathrm{C}_{\mathbf{m}}$ are respectively integrated data covariance and model covariance expressed as

$$\begin{aligned}\mathbf{C_d} &= \operatorname{diag}[\mathbf{C}_{\mathbf{d}1}, \mathbf{C}_{\mathbf{d}2}, \cdots],\\ \mathbf{C_m} &= \operatorname{diag}\left[\alpha_1^{-1}\mathbf{C}_{\mathbf{m}1}, \alpha_2^{-1}\mathbf{C}_{\mathbf{m}2}, \cdots\right],\end{aligned}$$

which are block diagonal matrices.

The τ is the column vector of

$$\tau = \begin{bmatrix}\vdots\\ \nabla\mathbf{m}_j \times \nabla\mathbf{m}_l\\ \vdots\end{bmatrix}, \forall j \neq l.$$