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Comparison between staggered grid finite–volume and edge–based finite–element modelling of geophysical electromagnetic data on unstructured grids

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This study compares two finite–element (FE) and three finite–volume (FV) schemes which use unstructured tetrahedral grids for the modelling of electromagnetic (EM) data. All these schemes belong to a group of differential methods where the electric field is defined along the edges of the elements. The FE and FV schemes are based on both the EM–field and the potential formulations of Maxwell's equations. The EM–field FE scheme uses edge–based (vector) basis functions while the potential FE scheme uses vector and scalar basis functions. All the FV schemes use staggered tetrahedral–Voronoï grids. Three examples are used for comparisons in terms of accuracy and in terms of the computation resources required by generic iterative and direct solvers for solving the problems. Two of these examples represent survey scenarios with electric and magnetic sources and the results are compared with those from the literature while the third example is a comparison against analytical solutions for an electric dipole source. Exactly the same mesh is used for all examples to allow for direct comparison of the various schemes. The results show that while the FE and FV schemes are comparable in terms of accuracy and computation resources, the FE schemes are slightly more accurate but also more expensive than the FV schemes.

1. Introduction

Common numerical methods use structured rectilinear grids for the modelling of geophysical electromagnetic (EM) data (e.g., Avdeev et al., 2002; Farquharson and Miensopust, 2011; Farquharson and Oldenburg, 2002; Fomenko and Mogi, 2002; Hursan and Zhdanov, 2002; Madden and Mackie, 1989; Mitsuhata and Uchida, 2004; Newman and Alumbaugh, 1995; Streich, 2009; Sugeng, 1998; Weiss and Constable, 2006; Weiss and Newman, 2002). While these methods are efficient in terms of accuracy and are simple to work with, they lack the flexibility required for representing arbitrary, complicated interfaces and for local refinement of the mesh. More flexibility is offered by the finite–volume (FV) and finite–element (FE) methods which can use unstructured grids. In unstructured grids, the facets of the elements conform to the irregular interfaces and, hence, avoid further refinements at these interfaces. These grids also allow local refinement of the mesh at observation points or at locations of high solution curvature (e.g., at the source in total–field approaches).

The FV method directly discretizes the integral form of the governing equations while the FE method approximates the solution of the weak form of a partial differential equation by minimizing an error function. Compared to the FE techniques, the FV methods are generally simpler in idea and more physically meaningful (see, e.g., Hirsch, 2007; Hermeline, 2009; Hermeline et al., 2008). The more rigorous mathematical background of the FE method, on the other hand, provides better control over the scheme. Moreover, unstructured FE schemes are usually more convenient for programming because, unlike FV discretizations, FE discretizations do not deal with complex geometrical structures in the mesh. For these reasons, the FE technique has been favored in geophysics by researchers who have worked with unstructured grids (e.g., Badea et al., 2001; Puzyrev et al., 2013; Schwarzbach et al., 2011; Um et al., 2012a,b).

Different classes of FE and FV methods have been used for solving EM problems on unstructured grids. Methods which solve for the electric and/or magnetic fields are called EM–field methods, and those which solve for the electric and magnetic potentials are commonly called potential (or A–ϕ) methods. (The main reason for using the potential method is the better conditioning of the problems in this method which suits iterative solvers.) FE methods use nodal–based and/or edge–based (vector) basis functions. Nodal basis functions were used by, e.g., Badea et al. (2001), Key and Ovall (2011), Li and Key (2007) and Puzyrev et al. (2013). Nédéléc (1980)

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introduced vector basis functions which were used by, e.g., Börner et al. (2008), Schwarzbach et al. (2011), Um et al. (2010) and Um et al. (2013). These basis functions have the advantage of allowing the discontinuity of the normal component of electric field at conductivity contrasts, and also of being divergence free (inside source/charge free elements).

There are also different classes of FV methods for unstructured grids. In the cell–centred FV methods, all the fields are co–located at the centres of the control–volumes (e.g., Remaki, 2000; Shankar et al., 1990). In the staggered methods, the unknowns are located at separate locations: Madsen and Ziolkowski (1990) employed a staggered grid in which the full 3–D vectors of electric and magnetic fields are defined on the edges of the primary and dual cells, respectively (Weiss, 2010, and Jahandari and Farquharson, 2014, 2015, belong to this category). Yee and Chen (1994, 1997) proposed a slightly different approach by locating the electric and magnetic fields at the vertices of primary and dual cells. In terms of the location of the unknown fields, cell–centred FV schemes can be seen as counterparts of nodal-based FE schemes (e.g., a cell–centred FV scheme with Voronoï cells as control–volumes corresponds to a nodal–based FE scheme on a tetrahedral mesh), and staggered FV schemes as equivalents of edge–based FE schemes.

During recent years, we developed several FV and FE schemes for the modelling of EM data on tetrahedral grids (Ansari and Farquharson, 2014; Jahandari and Farquharson, 2014, 2015). The fact that all these schemes belong to classes of FE and FV techniques where the unknown vector fields are defined at the edges of the elements motivated the comparison of these schemes. The EM–field FE scheme uses vector basis functions while the potential FE scheme uses both vector and scalar basis functions (Ansari, 2014). The EM–field and potential FV schemes all use staggered tetrahedral–Voronoï grids (Jahandari and Farquharson, 2014, 2015). In the EM–field FV and FE schemes, the unknown fields are defined along the edges of the tetrahedra while in the potential FV and FE schemes, the vector and scalar potentials are defined along the edges and at the nodes of the tetrahedra, respectively. There is only one FE potential scheme while there are two potential FV schemes (gauged and ungauged). Therefore, there are five frequency–domain total–field schemes (two FE and three FV). In the following sections, the theoretical background of the methods is briefly described and then three examples are used for comparing the schemes in terms of accuracy and in terms of computational resources required by generic direct and iterative solvers for solving the problems.

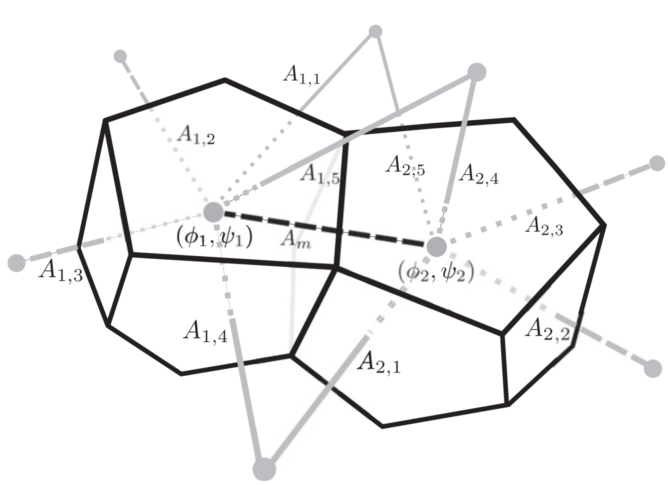

Fig. 1. The unknown vector and scalar potentials involved in approximating relation (10) for a tetrahedral edge. This edge is shown by a black dashed line and the unknown potential at this edge is A_m. The edges that are used for approximating the first term in this relation are shown by gray solid lines. All the edges in gray are used by the second term of this relation for approximating $\nabla\psi$.

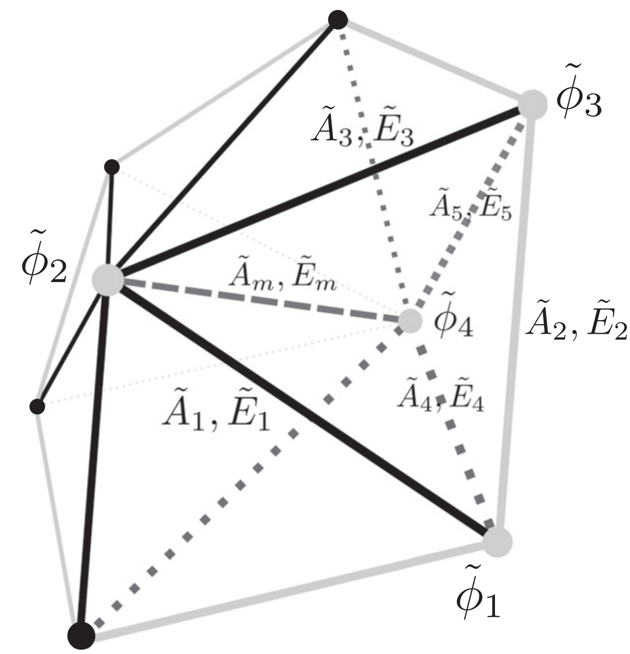

Fig. 2. The nodes and edges involved in discretizing relations (17) and (20) for the edge m (shown as a dashed line). As an example, the unknowns ($\tilde{E}$, $\tilde{A}$ and $\tilde{\phi}$) for one of the tetrahedra that share m are shown (the vertices of this tetrahedron are in gray).

2. Theory

In the direct EM–field method the problem is formulated in terms of electric and magnetic fields (**E** and **H**) while in the potential method, these fields are replaced by their representations in terms of magnetic vector potential and electric scalar potential (**A** and ϕ):

$$\mathbf{E} = -i\omega\mathbf{A} - \nabla\phi \tag{1}$$

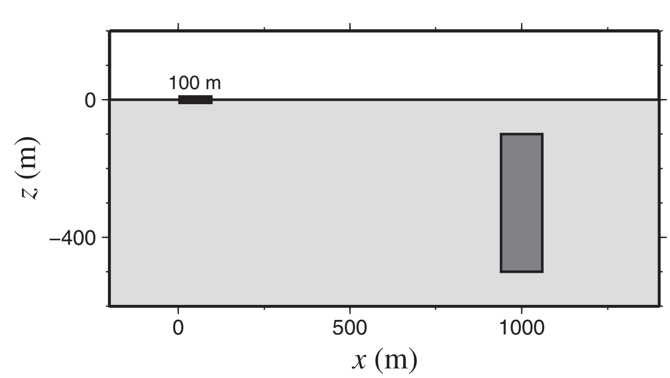

Fig. 3. A vertical section through the model in the first example. It shows the location of the 100m line source (the thick black segment) and the anomalous region (dark gray).