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2-D magnetotelluric modeling using finite element method incorporating unstructured quadrilateral elements

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ABSTRACT

In this research, the finite element (FE) method incorporating quadrilateral elements for solving 2-D MT modeling was presented. The finite element software was developed, employing a paving algorithm to generate the unstructured quadrilateral mesh. The accuracy, efficiency, reliability, and flexibility of our FE forward modeling are presented, compared and discussed. The numerical results indicate that our FE codes using an unstructured quadrilateral mesh provide good accuracy when the local mesh refinement is applied around sites and in the area of interest, with superior results when compared to other FE methods. The reliability of the developed codes was also confirmed when comparing both analytical solutions and COMMEMI2D model. Furthermore, our developed FE codes incorporating an unstructured quadrilateral mesh showed useful and powerful features such as handling irregular and complex subregions and providing local refinement of the mesh for a 2-D domain as closely as unstructured triangular mesh but it requires less number of elements in a mesh.

1. Introduction

From the past to the present, two-dimensional magnetotelluric modeling was successfully solved by various numerical techniques such as the finite difference (FD) method (Rao and Babu, 2006) and finite element (FE) method (Reddy and Rankin, 1975; Wannamaker et al., 1986; Key and Weiss, 2006; Franke et al., 2007; Lee et al., 2009). The efficiency and accuracy of the FD method were proven for a simple 2-D model (Rao and Babu, 2006). With the limitation of rectangular grids, many complex subregions in the 2-D domain such as modeling topography, irregular anomalous bodies, and seafloor cannot be handled by its discretization. Therefore, the solution obtained by the FD method may be inaccurate and unrelialable.

The finite element (FE) method is another numerical technique that is often used for 2-D MT forward modeling. Its efficiency and accuracy have been proven (Reddy and Rankin, 1975; Wannamaker et al., 1986; Key and Weiss, 2006; Franke et al., 2007; Lee et al., 2009). Unlike the FD method, the FE method is more flexible on discretization. Various mesh schemes such as structured triangular meshes (Wannamaker et al., 1986), unstructured triangular meshes with adaptive refinement (Franke et al., 2007; Key and Weiss, 2006) and structured quadrilateral meshes (Reddy and Rankin, 1975; Lee et al., 2009) can be used in the FE method. The structured triangular and quadrilateral mesh algorithms are simple and can handle domains with complex subregions, but the local refinement mesh feature cannot be implemented efficiently. In contrast, the unstructured triangular mesh algorithm can overcome this problem. However, it needs to incorporate an adaptive scheme to improve its accuracy and it requires a large number of triangular elements in complex subregions (Key and Weiss, 2006; Franke et al., 2007).

Presently, implementations of unstructured quadrilateral meshes already in the FE method are popular as discussed by Blacker and Stephenson (1991); Bern (1997); Owen (1998); Sarrate and Huerta (2000); Remacle et al. (2012). The advantages of unstructured quadrilateral mesh algorithms over triangular mesh algorithms such as accuracy, and computational resources have been extensively surveyed and discussed (Owen, 1998). So, it is both interesting and challenging to study and apply this type of algorithm to develop alternative FE methods to solve a 2-D magnetotelluric problem.

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In this work, we applied the FE method with an alternative unstructured quadrilateral mesh to solve a 2-D magnetotelluric problem. An unstructured quadrilateral mesh was used for our FE codes and was generated using the paving algorithm (Blacker and Stephenson, 1991). Here, the paving algorithm is reviewed and summarised. Suitable FE codes with the selected mesh type were developed and tested. The resulting numerical solution was validated and compared to both an analytical solution and other numerical methods. Furthermore, the capabilities of the alternative mesh algorithm to handle the 2-D domain are presented and discussed.

2. Governing equations

Magnetotelluric (MT) modeling describes the phenomena of the natural electromagnetic (EM) induction in the Earth. The naturally occurring EM fields act like a plane-wave with harmonic diffusion, the displacement currents can be neglected and the time-dependence is assumed to be $e^{-i\omega t}$, where ω is the angular frequency. For the case where the electrical conductivity σ is varied in only the x and z-directions, i.e., $\sigma = \sigma(x,z)$, the second-order partial equations for the 2-D MT problem with the bounded region $\Omega \subset \mathbb{R}^2$ is given by

$$\frac{\partial}{\partial x}\left(\alpha\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial z}\left(\alpha\frac{\partial u}{\partial z}\right) + \beta u = 0. \tag{1}$$

The notations α, β and u denote different representations depending on the two MT polarizations:

$$E\text{-polarization:} \quad u = E_y, \quad \alpha = 1, \quad \beta = i\omega\mu\sigma, \tag{2}$$

$$H\text{-polarization:} \quad u = H_y, \quad \alpha = 1/\sigma, \quad \beta = i\omega\mu, \tag{3}$$

where E_y and H_y are the strike aligned electric and magnetic fields, respectively, μ is the magnetic permeability in free space and $\mu = \mu_0 = 4\pi \times 10^{-7}$ $(V\,s/Am)$. The bounded region Ω is defined by $\Omega = \Omega_1 \cup \Omega_2 \cup \Gamma \cup \Gamma^{int}$, where Ω_1 is the air subregion, Ω_2 is the earth subregion, Γ is the outer boundary and Γ^{int} is the air-earth interface where electrical conductivity σ is discontinuous. The partial differential Eq. (1) is subjected to the Dirichlet boundary conditions

$$u = u_0(x,z) \quad \text{on} \quad \Gamma, \tag{4}$$

where $u_0(x,z)$ is obtained by solving the one-dimensional MT problem.

3. Mesh algorithm

To solve Eq. (1) for the 2-D magnetotelluric forward modeling subjected to the boundary condition as in Eq. (4) using the finite element method, discretization or mesh generation is the first and important step. For the unstructured quadrilateral mesh, a continuous 2-D domain, Ω, was meshed into many subdomains or elements. Here, $\Omega = \cup_{e=1}^{M}\Omega^e$, where Ω^e is the e-th element with a quadrilateral shape and M is the total number of elements. Presently, there are two categories of generating unstructured quadrilateral mesh: direct and indirect approaches (Owen, 1998). The direct approach generates the mesh from the given domain whereas the indirect approach generates a quadrilateral mesh from an existing triangular mesh by splitting or merging elements. The algorithms of some indirect approaches provide elements of poor quality and do not guarantee complete transformation of triangular elements to quadrilateral elements (Owen, 1998). In contrast, direct approaches guarantee that the generated meshes are composed entirely of quadrilateral elements. However, some of them may produce low-quality elements near complex boundaries of the domain and are unable to completely satisfy some constraints (Remacle et al., 2012). For these reasons, a direct approach was used for generating an unstructured quadrilateral mesh for the 2-D MT model employing the paving algorithm of Blacker and Stephenson (1991).

An example of the paving algorithm for 2-D magnetotelluric model is illustrated in Fig. 1.

At the beginning, the nodes are paved on the boundary of the domain (Fig. 1b). The distance between two nodes does not need to be uniform. Usually, the connectivities of nodes on the exterior Γ and interior Γ^{int} boundaries are set as permanent, i.e., the coordinate and number of node on these boundaries do not change. With permanent boundaries, an exterior paving boundary is paved counterclockwise and progresses inward from the exterior boundary. In contrast, the interior paving boundary is paved clockwise and progresses outward from the interior permanent boundary. This step is called row choice (Fig. 1c). Next, new rows are added and adjusted to correct the element size and improve the mesh quality by using

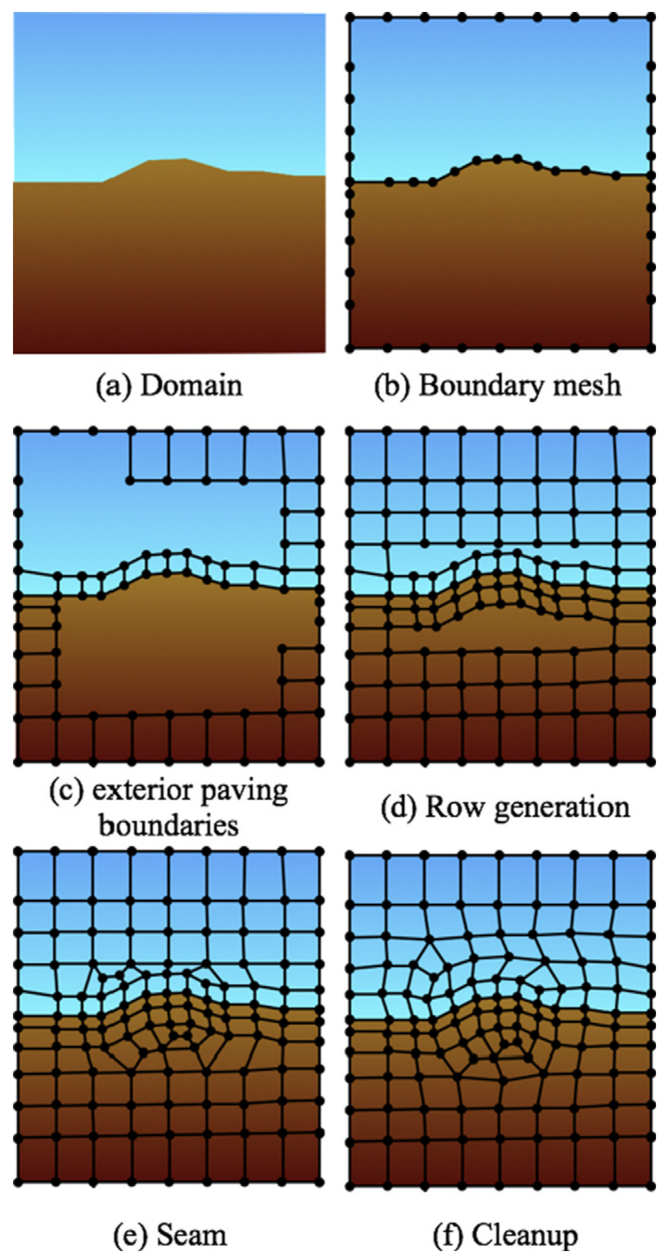

Fig. 1. Example of paving algorithm for 2-D model.