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2-D inversion of P-wave polarization data to obtain maps of velocity gradient

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ABSTRACT

Gradient mapping is a technique employed in the interpretation of tomographic velocity images for delineating geological structures. In this paper, a tomographic method is proposed for determining relative velocity gradient field from seismic polarization directions. This inverse problem is iteratively resolved by the damped least squares method. With Hamiltonian formulation of ray theory and under the assumption that the medium is weakly inhomogeneous, the problem formulation for polarization direction is approximately expressed as a function of relative velocity gradient. Explicit expressions of the Frechet derivatives of polarization directions with respect to model parameters are given. The proposed tomographic method is illustrated by conducting synthetic experiments for showing the ability of our method to recover relative velocity gradient field as well as its potential applicability to complex media. The test results demonstrate that the proposed method is a promising approach for imaging geological structures.

1. Introduction

Gradient mapping is commonly used for analyzing potential field data (Boschetti, 2005; Hsu et al., 1996; Métivier et al., 2016; Oruç et al., 2013; Sharpton et al., 1987; Zuo and Hu, 2015), which presents the data in a form that helps us to focus on the main features of potential field data and to interpret them in terms of geological structures. Stewart Fishwick introduced it as a tool employed in the interpretation of seismic velocity images obtained from traveltime tomographic inversion for delineating geological structures (Fishwick, 2006). Regions of strong gradient indicate rapid changes in velocity field, which generally imply the existence of geological boundaries such as contacts or faults. Gradient maps give information on the sharpness and continuity of geological boundaries, which is not easily obtained from tomographic velocity images (Fishwick, 2006; Schmandt, 2012; Wang et al., 2008).

To date seismic velocity gradient maps have no choice but to be translated from traveltime inversion results by a finite difference approximation (Fishwick, 2006; Ramachandran, 2012), so the reliability of gradient maps depends on that of traveltime inversion results. Traveltime is stationary with respect to perturbation in the ray path. It makes traveltime data insensitive to the exact position of velocity heterogeneities (Hu and Menke, 1992; Song et al., 2001). Sometimes the rays employed in inversion may deviate from the real paths, which results in a biased data kernel and thus misdistributes the velocity anomalies in the imaged velocity structure (Hu and Menke, 1992). In such a case, the gradient maps obtained from the resulting velocity model will appear some deviation.

Polarization (the slowness vector of a ray in our case) is a basic property of seismic waves, which is another source of data for seismic tomography and can be obtained easily from multicomponent seismic data (Falsaperla et al., 2002; Jurkevics, 1988; Perelberg and Hornbostel, 1994). Polarization data (the measurements of slowness vectors) have been utilized to be inverted for velocity models independently or simultaneously with other kinds of data, such as traveltimes (Bégat and Farra, 1997; Hu and Menke, 1992; Hu et al., 1993; Hu et al., 1994).

Polarization is fundamentally distinct from traveltime, which is sensitive to the velocity gradient along the ray path, and can probe velocity gradient (Hu and Menke, 1992; Song et al., 2001). However, there are no articles that have explicitly addressed velocity gradient inversion with polarization data. In this paper, we develop a tomographic method for recovering relative velocity gradient field directly from polarization direction data. This method is demonstrated through two validation tests for 1-D and 2-D models.

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2. Method

2.1. Relation between polarization direction and relative velocity gradient

Seismic tomography is a data inference technique for reconstructing the model of the Earth's interior, which can be seen as a way to find the model that best explains observed data with a supposed physical relationship between seismic data and seismic model. The relationship is used for solving the forward problem in tomographic inversion. In this section, we derive an approximate relationship between polarization direction and relative velocity gradient from ray theory in weakly inhomogeneous media.

Let's introduce the Hamiltonian function (Farra and Madariaga, 1987; Lambaré et al., 1996)

$$H(\mathbf{r},\mathbf{p},t) = \frac{1}{2}[\mathbf{p}^2 v^2(\mathbf{r}) - 1], \quad (1)$$

where t denotes the time abscissa along the ray trajectory, $\mathbf{r}$ the position, and $\mathbf{p}$ the slowness vector such that $\mathbf{p} = \nabla t(\mathbf{r})$ for ray trajectories. A ray is defined by its canonical vector $\mathbf{y}(t) = (\mathbf{r}(t),\mathbf{p}(t))$. This canonical vector of the rays satisfies Hamilton's equations (Billette and Lambaré, 1998)

$$\begin{aligned} \frac{\partial \mathbf{r}}{\partial t} &= \nabla_{\mathbf{p}} H = v^2(\mathbf{r})\mathbf{p} \\ \frac{\partial \mathbf{p}}{\partial t} &= -\nabla_{\mathbf{r}} H = -\mathbf{p}^2 v(\mathbf{r})\nabla v(\mathbf{r}) \end{aligned}, \quad (2)$$

with the initial condition $H(\mathbf{x},\mathbf{p},t) = 0$, where $\nabla_{\mathbf{p}}$ and $\nabla_{\mathbf{r}}$ denote the gradients with respect to the vectors $\mathbf{r}$ and $\mathbf{p}$, respectively. For a given model and a source-receiver pair $\mathbf{r}_b$ and $\mathbf{r}_e$, ray tracing gives us the ray integrals

$$\begin{aligned} \mathbf{r}_e &= \mathbf{r}_b + \int_L v^2(\mathbf{r})\mathbf{p}\,\mathrm{d}t \\ \mathbf{p}_e &= \mathbf{p}_b - \int_L \mathbf{p}^2 v(\mathbf{r})\nabla v(\mathbf{r})\,\mathrm{d}t \end{aligned}. \quad (3)$$

The integrals are taken along the ray L connecting the source position $\mathbf{r}_b$, and the receiver position $\mathbf{r}_e$ with the start-time and the end-time of arc $t = 0$ and $t = T$. $\mathbf{p}_b$ and $\mathbf{p}_e$ are the P-wave slowness vectors at source and receiver, respectively. $\mathbf{r}_e$ and $\mathbf{p}_e$ depend on the ray L and the velocity field through the ray. In this paper, we take $\mathbf{p}_e$ as the P-wave polarization vector and simply call it polarization as Hu and Menke did (Hu and Menke, 1992).

In 2-D media, the P-wave polarization direction is determined by the two components of polarization vector in horizontal and vertical directions. We decompose the polarization vector $\mathbf{p}_e$ into the two components

$$\begin{aligned} p_e^x &= p_b^x - \int_L \mathbf{p}^2 v(\mathbf{r})\nabla^x v(\mathbf{r})\,\mathrm{d}t \\ p_e^z &= p_b^z - \int_L \mathbf{p}^2 v(\mathbf{r})\nabla^z v(\mathbf{r})\,\mathrm{d}t \end{aligned}, \quad (4)$$

where the superscript x and z indicate that the corresponding components of the vectors is in x and z directions, respectively. We take the incidence of seismic ray at receiver as P-wave polarization direction, which is measured from the vertical direction. With Eq. (4) and by introducing the arc length of the ray s as the integral variable, we obtain the relationship between polarization direction and velocity model

$$\tan(\beta) = \frac{\sin(\alpha) - \int_L \frac{\nabla^x v(\mathbf{r}) v(\mathbf{r}_b)}{v^2(\mathbf{r})}\mathrm{d}s}{\cos(\alpha) - \int_L \frac{\nabla^z v(\mathbf{r}) v(\mathbf{r}_b)}{v^2(\mathbf{r})}\mathrm{d}s}, \quad (5)$$

where $\mathrm{d}s = v(\mathbf{r})\mathrm{d}t$, is the differential of the arc length of the ray, α and β are the angles between slowness vector of the ray and vertical direction at source and receiver, respectively. β is also taken as the polarization direction at receiver.

We expand the integrand in the numerator on the right side of Eq. (5) around $\mathbf{r}_b$ as a taylor series

$$\begin{aligned} \frac{\nabla^x v(\mathbf{r}) v(\mathbf{r}_b)}{v(\mathbf{r})^2} = {} & \frac{\nabla^x v(\mathbf{r}_b)}{v(\mathbf{r}_b)}\frac{v(\mathbf{r}_b)}{v(\mathbf{r}_b)} + \frac{v(\mathbf{r}_b)}{v(\mathbf{r}_b)}\left(\frac{\nabla^x v(\mathbf{r})}{v(\mathbf{r})} - \frac{\nabla^x v(\mathbf{r}_b)}{v(\mathbf{r}_b)}\right) \\ & + \frac{\nabla^x v(\mathbf{r}_b)}{v(\mathbf{r}_b)}\left(\frac{v(\mathbf{r}_b)}{v(\mathbf{r})} - 1\right). \end{aligned} \quad (6)$$

In media with low contrast velocity, the difference between $v(\mathbf{r}_b)$ and $v(\mathbf{r})$ are small, so the third term in Eq. (6) can be ignored, and then Eq. (6) is now approximated as

$$\frac{\nabla^x v(\mathbf{r}) v(\mathbf{r}_b)}{v(\mathbf{r})^2} = \frac{\nabla^x v(\mathbf{r})}{v(\mathbf{r})}. \quad (7)$$

In the same way, we obtain

$$\frac{\nabla^z v(\mathbf{r}) v(\mathbf{r}_b)}{v(\mathbf{r})^2} = \frac{\nabla^z v(\mathbf{r})}{v(\mathbf{r})}. \quad (8)$$

Substituting the approximate expressions (7) and (8) into Eq. (5), we get

$$\tan(\beta) = \frac{\sin(\alpha) - \int_L \frac{\nabla^x v(\mathbf{r})}{v(\mathbf{r})}\mathrm{d}s}{\cos(\alpha) - \int_L \frac{\nabla^z v(\mathbf{r})}{v(\mathbf{r})}\mathrm{d}s}. \quad (9)$$

The integrands in Eq. (9) are the components of the vector $\frac{\nabla v(\mathbf{r})}{v(\mathbf{r})}$, which we call relative velocity gradient. This equation is concerning the relationship between relative velocity gradient $\frac{\nabla v(\mathbf{r})}{v(\mathbf{r})}$ and polarization direction β.

2.2. Model parameterization

Model parameterization is a main procedure in seismic tomographic inversion. We digitize the medium into a regular set of triangular cells (Song et al., 2001) and the relative velocity gradient values are specified at the grid points of the cells. Inside each triangular cell, we think the relative velocity gradient distribution varies linearly with the spatial coordinates x and z. Therefore, the relative velocity gradient field is continuous within the whole medium. With such digitization, the relative velocity gradient field in the medium can be conveniently represented as

$$\frac{\nabla v(\mathbf{r})}{v(\mathbf{r})} = \sum_{j=1}^{N} f_j(\mathbf{r})\mathbf{m}(j), \quad (10)$$

where $\mathbf{m}(j)$ is the relative velocity gradient value at the jth grid point and $f_i(\mathbf{r})$ are N dimensionless basis functions. The basis functions we use are defined as follows. Denoting the position vectors of the three vertices of a triangular cell as $\mathbf{I}_k$, $\mathbf{I}_p$, $\mathbf{I}_q$, then the basis function for the vertex k is obtained as (Song et al., 2001)

$$f_k(\mathbf{r}) = \begin{cases} \frac{1}{D}[|\mathbf{I}_p \times \mathbf{I}_q| + |\mathbf{I} \times (\mathbf{I}_p - \mathbf{I}_q)|] & \text{for } \mathbf{r} \text{ within the triangle} \\ 0 & \text{for } \mathbf{r} \text{ outside the triangle} \end{cases}, \quad (11)$$

where $\mathbf{l}$ is the position vector contained within the triangle and $D = |(\mathbf{I}_q - \mathbf{I}_k) \times (\mathbf{I}_q - \mathbf{I}_p)|$ is the area of a parallelogram whose two sides are $(\mathbf{I}_q - \mathbf{I}_k)$ and $(\mathbf{I}_q - \mathbf{I}_p)$. The basis functions corresponding to vertices p and q are obtained by permuting the indices in clockwise order of k, p, and q. These basis functions depend linearly on the position $\mathbf{r}$.

2.3. Solving the inverse problem

We take the least-squares criterion as a misfit function over the model space and try to find a model which minimizes the misfit function

$$C(\mathbf{m}) = \|\boldsymbol{\beta}_o - \boldsymbol{\beta}(\mathbf{m})\|_2^2, \quad (12)$$

where $\mathbf{m}$ is the vector of model parameters. $\boldsymbol{\beta}$ and $\boldsymbol{\beta}_o$ are the vectors of theoretical and observed data, respectively. For our method, the model is described by relative velocity gradient, the observed data are