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Constructing stochastic models for dipole fluctuations from paleomagnetic observations

Bruce Buffett [a,*], Abhijit Puranam [b]

[a] Department of Earth and Planetary Science, University of California, Berkeley, CA 94720, USA
[b] Division of Geological & Planetary Sciences, Caltech, Pasadena, CA 91125, USA

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ABSTRACT

Records of relative paleointensity are subject to several sources of error. Temporal averaging due to gradual acquisition of magnetization removes high-frequency fluctuations, whereas random errors introduce fluctuations at high frequency. Both sources of error limit our ability to construct stochastic models from paleomagnetic observations. We partially circumvent these difficulties by recognizing that the largest affects occur at high frequency. To illustrate we construct a stochastic model from two recent inversions of paleomagnetic observations for the axial dipole moment. An estimate of the noise term in the stochastic model is recovered from a high-resolution inversion (CALS10k.2), while the drift term is estimated from the low-frequency part of the power spectrum for a long, but lower-resolution inversion (PADM2M). Realizations of the resulting stochastic model yield a composite, broadband power spectrum that agrees well with the spectra from both PADM2M and CALS10k.2. A simple generalization of the stochastic model permits predictions for the mean rate of magnetic reversals. We show that the reversal rate depends on the time-averaged dipole moment, the variance of the dipole moment and a slow timescale that characterizes the adjustment of the dipole toward the time-averaged value. Predictions of the stochastic model give a mean rate of 4.2 Myr^{-1}, which is in good agreement with observations from marine magnetic anomalies.

1. Introduction

The spectrum of fluctuations in the geomagnetic dipole offers insights into the origin of the magnetic field and the dynamics of Earth's core (Constable and Johnson, 2005). Each distinct timescale bears the fingerprints of the underlying physical processes (e.g. Sakuraba and Hamano, 2007. Paleomagnetic observations are essential for characterizing the long-term behavior, yet no single source of information is sufficient to capture the full range of dynamics. Instead, we require an integrated approach to combine different types of measurements into a composite record that spans a broad range of timescales.

One important source of information comes from measurements of relative paleointensity in marine sediments (Valet, 2003). Records are stacked and calibrated using independent estimates of absolute paleointensity to produce models for the virtual axial dipole moment (VADM) over the past two million years (Valet et al., 2005; Ziegler et al., 2011). Sediments acquire a magnetization over several thousand years (Roberts and Winkholfer, 2004), so the true signal is averaged in time. Uncertainties in dating can have a similar affect because paleomagnetic records from different times may be stacked together.

Higher resolution records have been obtained for the past 10 kyr using a combination of archeomagnetic and lake sediment data. These data have improved spatial resolution, so the geomagnetic field can be expanded in low-degree spherical harmonics (e.g. Korte and Constable, 2011). Even higher resolution records are available from historical observations (Jackson et al., 2000). Taken together these records provide a comprehensive description of fluctuations in the dipole field, but the task of combining these results into a single coherent model is a challenge.

Stochastic models are a useful tool because they enable quantitative predictions over a range of timescales. This facility is important for combining different types of data with different levels of temporal resolution. There is also good reason to think that stochastic models can represent the relevant processes in the core. Stochastic models have been constructed from geodynamo simulations with only a few model parameters, yet these models are capable of reproducing most of the variability in these simulations (Kuipers et al., 2009; Buffett et al., 2014; Bouligand et al., 2016).

* Corresponding author.
E-mail address: bbuffett@berkeley.edu (B. Buffett).

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Synthetic studies using geodynamo simulations are an ideal test of the general approach because the simulations have relatively low numerical error and we can control the temporal resolution of the output. None of these advantages apply when we use paleomagnetic observations to construct stochastic models. Significant errors are present in the estimates of the dipole field, which affect the construction of the stochastic model. We also need to deal with temporal averaging because it limits our ability to sample the stochastic process. The goal of this study is to address the practical limitations of dealing with paleomagnetic observations and to devise a strategy for constructing models that can explain both paleomagnetic and historical records. We focus primarily on the power spectrum of dipole fluctuations, although we find that the resulting stochastic models can also account for the observed reversal rate and the duration of polarity transitions.

2. Stochastic description of dipole fluctuations

Stochastic models were introduced by Langevin (1908) to describe Brownian motion. A small particle in water was assumed to move under the combined influence of viscous resistance and a random force due to collision with (unseen) water molecules. The viscous force was treated as a slowly varying deterministic quantity, whereas the force due to collisions with water molecules was treated as a rapidly fluctuating random process.

Brownian motion serves as a loose analogy for the evolution of the geomagnetic dipole moment. The deterministic part of the dipole moment represents the opposing influences of dipole decay and the time-averaged dipole generation. Rapid fluctuations in dipole generation about the time average can be attributed to (unseen) turbulent flow, which we treat as a random process. We denote the axial dipole moment by $x(t)$ and describe its time evolution using a stochastic differential equation (Van Kampen, 1992)

$$\frac{dx}{dt} = v(x) + \sqrt{D(x)}\Gamma(t), \quad (1)$$

where the drift term, $v(x)$, describes the deterministic part of the evolution and the noise term, $D(x)$, defines the amplitude of the random part. The time dependence of the random process, $\Gamma(t)$, is assumed to be Gaussian with a vanishing time average

$$< \Gamma(t) >= 0. \quad (2)$$

We also assume that the correlation time of the noise source is short compared with the sampling of $x(t)$. In this case the autocovariance function of $\Gamma(t)$ can be approximated by a Dirac delta function,

$$< \Gamma(t_1)\Gamma(t_2) >= 2\delta(t_1 - t_2), \quad (3)$$

where the factor of two is a common convention (e.g. Risken, 1989).

Estimates for $v(x)$ and $D(x)$ can be extracted from a realization of the stochastic process (e.g. Friedrich et al., 2011). The drift term is defined by

$$< x(t + \Delta t) - x(t) >= v(x)\Delta t + O(\Delta t^2) \quad (4)$$

and the noise term can be approximated by

$$< [x(t + \Delta t) - x(t)]^2 >= 2D(x)\Delta t + O(\Delta t^2), \quad (5)$$

where the time averages are taken for a specific value of $x = x(t)$. In practice, the dipole moment is divided into a finite number of bins and a time average is evaluated for each bin. The time increment, Δt, is chosen to be long enough that $\Gamma(t)$ and $\Gamma(t + \Delta t)$ are uncorrelated, but short enough that higher order terms in Δt are small enough to neglect.

Applying (4) and (5) to the output of a geodynamo model (Buffett et al., 2014; Meduri and Wicht, 2016) shows that the drift term, $v(x)$, is well represented by

$$v(x) = -\gamma(x- < x >), \quad (6)$$

where $< x >$ denotes the time average and γ is a constant that defines the inverse timescale for slow adjustments of the dipole. A similar representation for $v(x)$ has been recovered from VADM estimates (Brendel et al., 2007; Buffett et al., 2013). Very similar values for the constant, $\gamma \approx 0.034$ kyr^{-1}, were reported for the SINT-2000 model of Valet et al. (2005) and the PADM2M model of Ziegler et al. (2011). By comparison, the noise term, $D(x)$, has a weaker dependence on x. It suffices for our purposes to treat D as a constant and denote its value by D_{eq}.

Simple representations for the drift and noise terms permit closed-form solutions for the power spectrum of fluctuations about the time average (e.g. $\epsilon(t) = x(t)- < x >$). Defining the Fourier transform of $\epsilon(t)$ by

$$\epsilon(f) = \int_{-\infty}^{\infty} \epsilon(t) e^{-i2\pi ft}\, dt, \quad (7)$$

the power spectrum becomes (Buffett and Matsui, 2015)

$$S_\epsilon(f) = \frac{D_{eq}}{(\gamma^2 + 4\pi^2 f^2)} S_\Gamma(f), \quad (8)$$

where the power spectrum for a white noise source (with a variance of 2) is

$$S_\Gamma(f) = 2. \quad (9)$$

The theoretical spectrum in (8) agrees well with a direct calculation of the power spectrum from a geodynamo model (see Fig. 1). Departures at high frequency can be improved by allowing for the influence of correlated noise (Buffett and Matsui, 2015; Bouligand et al., 2016). The spectrum for $\epsilon(t)$ with correlated noise (denoted by $S^c_\epsilon(f)$) reduces the power at high frequencies, but it does not change the behavior at low frequencies. It is important to note that the drift and noise terms are recovered from the geodynamo model using (4) and (5) with a time difference of $\Delta t = 1$ kyr. No long-period information goes into the estimation of $v(x)$ and $D(x)$, yet

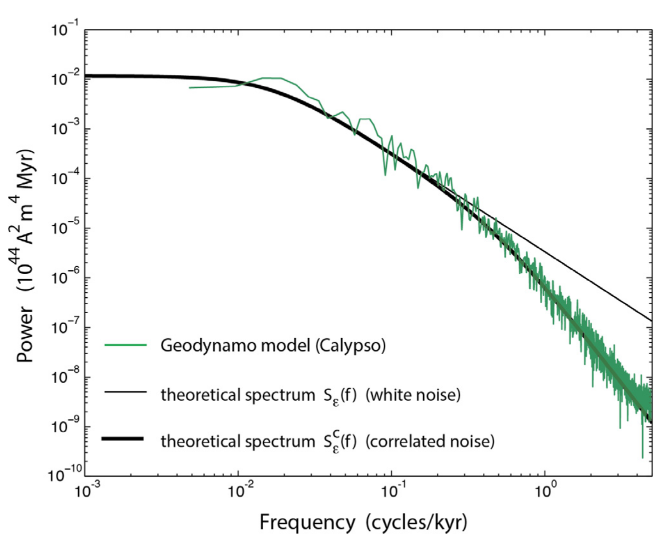

Fig. 1. A power spectrum of dipole fluctuations from a numerical geodynamo simulation (Matsui et al., 2014), compared to predictions from two stochastic models. One stochastic model assumes a white noise source and the other assumes correlated noise. Both models are capable of predicting the low-frequency fluctuations even though the drift and diffusion terms are constructed from short-period information.