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Three-dimensional stability analysis of slopes in hard soil/soft rock with tensile strength cut-off

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ABSTRACT

Many collapse analyses of slopes in soils or soft rocks use the classical Mohr-Coulomb yield function to define the strength of geomaterials. In the presence of bonded particles and grains, this function predicts uniaxial tensile strength and even greater isotropic tensile strength. Testing for material properties, however, is typically carried out in the compressive regime; the tensile strength is then burdened by uncertainties, as it is a result of extrapolation of test results into the tensile regime. A three-dimensional limit analysis of slopes is presented with the geomaterial described by a yield surface with tensile strength cut-off. The multiplicity of admissible collapse mechanisms is enriched, as the tension cut-off allows construction of mechanisms that include rupture modes. Stability factors for slopes with tensile strength cut-off are reduced compared to those based on the classical Mohr-Coulomb strength envelope, with the largest drop for steep slopes subjected to seepage. The stability factor for a 70-degree slope subjected to seepage can be reduced by as much as 69% when tensile strength cut-off is considered.

1. Introduction

Stability of both natural and anthropogenic slopes is a paramount problem in both geological and geotechnical engineering, with well-established literature. Most analyses include two-dimensional mechanisms of collapse with the soils' ability to resist yielding described by a linear strength envelope. Both of these assumptions are relaxed in this paper: three-dimensional mechanisms of slope collapse are considered and the soil strength is described by a nonlinear strength envelope, truncated in the tensile regime.

Two-dimensional analyses of slope stability are preferred by many, because of their relative simplicity and conservative outcome. However, most slope failures have three-dimensional features, supporting the motivation for the development of 3D analyses. In cases where the extent of the failure mass is restricted, for instance, by adjacent rock formation, a three-dimensional analysis may be called for. This is certainly the case in excavation slopes; three-dimensional analyses are also needed when back-calculating soil strength properties from known instabilities, where plane analyses may return estimates of strength that are not conservative.

Early observations of slope failures and their analyses all included two-dimensional mechanisms. Observations of failure in clay embankments led Collin (1846) to believe that the shape of the failure surface is a cycloid. Early analyses of stability of slopes were based on circular (cylindrical) and log-spiral failure surfaces (Fellenius, 1927; Taylor, 1937; Drucker and Prager, 1952). Early 3D analyses included undrained failures (Baligh and Azzouz, 1975; Gens et al., 1988) as the rotational mechanisms in incompressible materials are easier to achieve: any surface of revolution forms an admissible failure surface. The analyses in dilative materials relied on the extension of “slice” techniques to “column” techniques (Hovland, 1977; Hungr, 1987). A rather interesting approach to rotational failures was presented by Leshchinsky et al. (1985), who applied a variational approach to finding the critical failure surface. The kinematic approach of limit analysis with translational collapse mechanisms was applied in 3D analysis of slopes by Drescher (1983) and Michalowski (1989), and it was extended later to rotational collapse (de Buhan and Garnier, 1998; Michalowski and Drescher, 2009; Michalowski, 2010). The failure surface found by Leshchinsky et al. (1985) as a result of variational search (see also Zhang et al., 2016) appears to be a special case of what was postulated in limit analysis by Michalowski and Drescher (2009). The finite element approach to stability analysis brings the advantage of no need to predetermine the geometry characteristics of the failure surface (Griffiths and Marquez, 2007; Liu et al., 2017), but it comes with the issue of non-convergence at the instant of failure. The limit analysis approach using the finite element framework was found useful in addressing 3D stability (Li et al., 2010), and this approach can cope with complex boundary shapes and soil inhomogeneity more easily than the

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Fig. 1. Head scarp of the Twin Sisters Trail shallow landslide, Rocky Mountain National Park, Colorado (September 2013).

'analytical' approach of limit analysis can.

This paper focuses on 3D analysis of slope stability in hard soils and soft rocks with tensile strength cut-off. The motivation for the development of an analysis with tension cut-off stems from the uncertainties in the tensile strength determined from extrapolation of test results in the compressive regime. Tension cut-off introduces nonlinearity into the strength envelope, and allows for kinematic discontinuities with a large separation velocity relative to shear. This is consistent with observations of some landslides, which have very steep failure surfaces at the head of the slide, consistent with a mode of failure similar to toppling. One such example is illustrated in Fig. 1.

This shallow slide occurred in September of 2013 in a mountainous region, and intersected the trail leading to the Twin Sisters peaks in Rocky Mountain National Park, Colorado. The area was wooded, but the slide occurred beneath the tree roots, yet did not reach the bed rock; it was associated with substantial seepage, and the slide was approximately parallel to the surface. The width of the slide at its head was about 50 m, with an about-vertical failure surface, two-to-four meters high. To form a mechanism of collapse consistent with material translation parallel to the slope, the deformation at the steep head region of the slide likely involved a large separation component, driving the mass away from the material at rest, consistent with the presence of tension cut-off in the material's yield condition. Such deformation at the head of the slide can no longer be interpreted as the volumetric strain (or dilatancy), but rather a *rupture* of the material, more often seen in toppling failures. This conjecture is consistent with the presence of tensile strength cut-off in the yield function. Of course, one could also construct failure mechanisms that might leave a vertical head scarp without resorting to tension cut-off.

Not relying on the tensile strength in slope stability analyses was suggested earlier, but it was dealt with in a somewhat indirect manner. To avoid the presence of tensile stresses in the slope mass, a tension crack was introduced by Spencer (1968) (also, Duncan and Wright, 2005; Utili, 2013, and Michalowski, 2013). However, this is a very different approach from the one offered in this paper. An existing tension crack is part of the geometry of the boundary value problem, whereas the tension cut-off is part of the material model. The latter is likely to be more useful, and not only in limit analysis. For instance, to introduce an existing crack into the finite element analysis, its location needs to be defined first, whereas the method proposed here removes the tensile stresses from the slope by eliminating tension from admissible stresses defined by the strength envelope.

The investigation in this paper addresses the question whether the presence of tensile strength in the yield condition has an influence on the outcome of the 3D stability analysis of slopes, in particular, whether stability factors calculated based on the classical M-C yield condition overestimate those calculated without tensile strength. The novelty in this paper is in presenting a three-dimensional stability analysis with tension cut-off, demonstrating failure modes that involve *rupture* of the material, and demonstrating that steep slopes subjected to seepage are most vulnerable to tension cut-off.

2. Tensile strength cut-off

Often misinterpreted as a feature in the failure mechanism, tension cut-off is a material property. It is an integral part of the yield surface. The strength of rocks is typically described with non-linear yield envelopes (e.g., Hoek and Brown, 1980; Hoek et al., 2002), and the tensile strength is adjusted depending on the state of the rock. For example, for weathered rocks, the tensile strength is often reduced or even taken as zero. For bonded soils, a linear strength envelope in the compressive regime is used most often, extrapolated into the tensile regime, without consideration given to the presence or absence of true tensile strength. Drucker and Prager (1952) suggested eliminating the tensile strength from the limit stress envelope for soils, while Paul (1961) applied tension cut-off to the Mohr-Coulomb yield condition to consider the brittle behavior of rocks. This concept was used later in analyses of brittle materials (e.g., Chen and Drucker, 1969; Michalowski, 1985; Chen and Liu, 1990), and it is used in this paper to limit or eliminate tension from admissible stresses.

The yield surface for bonded geomaterial (e.g., soft rock) in the principal stress space is presented in Fig. 2(a). It is the Mohr-Coulomb surface with an additional three mutually perpendicular planes (ABCD being one of them) limiting the admissible stress states in the tensile regime, as proposed by Paul (1961). Limitation on tension so conceived is essentially the Galileo-Rankine tensile strength criterion; a smooth surface of this kind was discussed recently by Lagioia et al. (2014). A cross-section of the surface in Fig. 2(a) with a plane intersecting the triaxial compression ($\sigma_1 > \sigma_2 = \sigma_3$) and extension ($\sigma_2 = \sigma_3 > \sigma_1$) meridians is illustrated in Fig. 2(b), along with the octahedral cross-section. Every point in the space in Fig. 2(a) can be represented with three stress circles on the σ-τ plane in Fig. 2(c). For example, the stress state at point K($\sigma_1, \sigma_2, \sigma_3$) is represented by the three circles shown in Fig. 2(c). Section SP of the limit circle C_3 constitutes a segment of the strength envelope. Each point on the envelope represents components of a traction vector on a failure surface in a collapse mechanism, whereas $\mathbf{v}$ depicts the velocity discontinuity vector. As the normality flow rule is enforced in limit analysis, points on section SP allow deformation with large normal components relative to shear, with angle δ varying from ϕ at point S to 90° at point P. Angle δ is not a parameter defining dilatancy in the Reynolds (1885) sense. The nature of the deformation process associated with tension cut-off was not intended by Paul (1961) to be continual strain, but fracture. Hence, more appropriately, this deformation should be interpreted as material rupture and separation (rather than ductile strain associated with dilatancy), and angle δ referred to as the *rupture angle*. Deformation with rupture angle δ in the range of ϕ to 90° is represented in Fig. 2(a) by a fan of strain rate vectors at point K, contained by two limiting directions marked as s and p. It will be emphasized later in Section 3 that deformation with large angle δ does not relax the rigor of limit analysis, and, from the plasticity standpoint, the result is still a strict bound on the true solution.

Soils that are not bonded have neither tensile strength nor uniaxial compressive strength, even if substantial dilatancy is derived from grain and particle interlocking, but a bonded geomaterial can exhibit considerable uniaxial compressive strength and some tensile strength. However, as mentioned earlier, tensile strength is not a subject of routine testing; instead, the tensile strength is an outcome of extrapolating the test results from the compressive regime. This paper examines what the influence of reducing or eliminating tensile strength from the strength envelope is on the outcome of stability analyses. A