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Probabilistic transformation models for preconsolidation stress based on clay index properties

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ABSTRACT

This paper develops probabilistic transformation models that predict the probability density function (PDF) of the preconsolidation stress (σ'_p) of a clay based on its index properties. These probabilistic transformation models are more versatile than traditional transformation models that only provide point estimates, because the PDF can quantify the transformation uncertainty in σ'_p and also can be further used to develop median and confidence interval estimates for σ'_p. The confidence interval is especially useful because the transformation uncertainty in σ'_p is fairly significant. Three probabilistic transformation models are developed: one is a generic model and the other two are models specialized for contractive and dilative clays. The generic model is applicable to both contractive and dilative clays, but its transformation uncertainty is larger. The two specialized models require the prior knowledge about the contractive/dilative behavior for the clay of interest, but their transformation uncertainties are smaller. The performances of the three probabilistic transformation models are verified by statistical cross-validation and independent validation database. The analytical forms for the median and confidence interval estimates are presented in this paper so that engineers do not need to derive the Bayesian equations.

1. Introduction

The preconsoilidation stress (σ'_p) and overconsolidation ratio (OCR) are important parameters that quantify the stress history of a clay. The undrained shear strength of a clay can be estimated based on its σ'_p (Mesri, 1975, 1989). The at-rest earth pressure coefficient can be estimated based on its OCR (Mayne and Kulhawy, 1982). The contractive/dilative behavior of a clay during shearing also depends on OCR. σ'_p for a clay can be determined in laboratory using the oedometer test. It can be also estimated based on the index properties of a clay through a transformation model (Stas and Kulhawy, 1984; Nagaraj and Srinivasa Murthy, 1986; DeGroot et al., 1999; Ching and Phoon, 2012; Kootahi and Mayne, 2016) as a first-order approximation. The latter (index properties) is desirable for the preliminary design stage where extensive laboratory tests are not yet conducted and for scenarios where the site investigation budget is limited.

However, there is significant uncertainty in the estimated σ'_p given the knowledge of the index properties. Table 1 shows the bias and coefficient of variation (COV) for several σ'_p transformation models based on index properties. The bias is defined as the mean value for (measured σ'_p)/(predicted σ'_p) and COV is the coefficient of variation for (measured σ'_p)/(predicted σ'_p). The biases and COVs for the transformation models in Table 1 are calibrated by the CLAY/10/7490 database developed in Ching and Phoon (2014a). The magnitude for the transformation uncertainty is quantified by the COV. Consider the most recent transformation model developed by Kootahi and Mayne (2016). It has the smallest COV and yet the COV = 0.67 is still fairly large. The COVs for other models are even larger.

It is therefore desirable to quantify the significant transformation uncertainty in σ'_p by adopting a probabilistic transformation model. The main difference between probabilistic and traditional transformation models is that the former provides a probability density function (PDF) for σ'_p, whereas the latter only provides a point estimate for σ'_p. The availability of a PDF is a significant advantage because design engineers can get a sense for the magnitude of the transformation uncertainty, e.g., the 95% confidence interval for σ'_p is the interval bounded by the 0.025- and 0.975-fractiles for the PDF. Point estimates can also be derived from the PDF, e.g., the mean or median estimate; Eurocode 7 has suggested the 0.05-fractile as the characteristic value (CEN 2004). More recently, probabilistic transformation models for some soil and rock properties have been proposed (Yan et al., 2009; Wang et al., 2010; Ching and Phoon, 2012, 2014b; Wang and Cao, 2013; Ching et al., 2014, 2017b; D'Ignazio et al., 2016; Feng and Jimenez 2015; Ng et al., 2016), but to the authors' best knowledge, the

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Table 1
Biases and coefficients of variation (COVs) for several σ'_p transformation models.

Literature	Transformation model	Calibration results		
		n	Bias	COV
Stas and Kulhawy (1984)	$\sigma_p'/P_a \approx 10^{1.11-1.62\times LI}$ (for $S_t < 10$)	248	1.13[a]	1.36[a]
Nagaraj and Srinivasa Murthy (1986)	$\sigma_p'(kPa) \approx 10^{5.97-5.32\times(w_n/LL)-0.25\times\log_{10}[\sigma_{v0}'(kPa)]}$	1242	1.38[a]	3.46[a]
DeGroot et al. (1999)	$\sigma_p'(kPa) \approx 10^{2.9-0.96\times LI}$	1313	1.86[a]	1.20[a]
Ching and Phoon (2012)	$\sigma_p'/P_a \approx 0.235\times LI^{-1.319}\times S_t^{0.536}$	506	1.36	0.89
Kootahi and Mayne (2016)	$\sigma_p'\big/P_a \approx \begin{cases}1.62\times(\sigma_{v0}'/P_a)^{0.89}(LL)^{0.12}(w_n)^{-0.14} & \text{if } DS^b > 1.123\\ 7.94\times(\sigma_{v0}'/P_a)^{0.71}(LL)^{0.53}(w_n)^{-0.71} & \text{if } DS \le 1.123\end{cases}$	1242	1.10	0.67

P_a = one atmosphere pressure; LI = liquidity index; S_t = sensitivity; w_n = natural water content; LL = liquid limit; σ'_{v0} = in-situ effective vertical stress; n = number of calibration cases.
[a] The ratios (measured σ'_p)/(predicted σ'_p) for some cases are extremely large, so an alternative additive definition (Ching et al., 2017a) for bias and COV is adopted: bias = (sample mean of measured σ'_p) / (sample mean of predicted σ'_p), standard deviation = sample standard deviation of measured σ'_p − bias × predicted σ'_p. The COV = (standard deviation) / (sample mean of measured σ'_p).
[b] $DS = 5.152\times\log_{10}(\sigma'_{v0}/P_a) - 0.061\times LL - 0.093\times PL + 6.219\times e_n$, where e_n is the natural void ratio. It may be estimated as $e_n \approx w_n/2.65$.

probabilistic transformation model for σ'_p based on clay index properties is not yet available.

This paper develops probabilistic transformation models for σ'_p based on the index properties of a clay. First, the multivariate PDF for (σ'_p/P_a, σ'_{v0}/P_a, PL, LL, w_n) is constructed from a multivariate clay database using the translation approach (Liu and Der Kiureghian, 1986; Li et al., 2012), where P_a = 101.3 kN/m^2 is one atmosphere pressure, PL and LL are plastic and liquid limits, and w_n is the natural water content. σ'_{v0} is considered because it is well known that σ'_p is positively correlated to σ'_{v0}. w_n is considered because a large σ'_p tends to produce a smaller void ratio, hence a smaller water content. PL and LL are considered because they are limiting values for w_n. (PL, LL, w_n) can be consolidated into a single variable LI, liquidity index. However, it is found in this paper that the prediction accuracy for σ'_p will decrease if such consolidation is adopted. As a result, (PL, LL, w_n) are not consolidated into LI.

The constructed multivariate PDF for (σ'_p/P_a, σ'_{v0}/P_a, PL, LL, w_n) serves as the prior PDF for the subsequent Bayesian analysis. Then, given the site-specific information on (σ'_{v0}/P_a, PL, LL, w_n) for a clay of interest, the PDF of its σ'_p/P_a can be updated by the Bayesian analysis. The entire framework has the numerical advantage that the updated PDF of σ'_p/P_a and its point and interval estimates can be expressed in analytical forms. Design engineers can directly implement these analytical equations without the need to re-derive the Bayesian equations. The performance of the resulting probabilistic transformation models will be compared with existing traditional transformation models. The effectiveness of the probabilistic transformation models will be assessed by validation databases.

2. Multivariate clay database

The CLAY/10/7490 database (Ching and Phoon, 2014a) is a generic (global) clay database consisting of 7490 data points from 30 countries/regions worldwide. The clay properties cover a wide range of OCR, sensitivity (S_t), and plasticity index (PI). In the database, there are a subset of 1217 data points with simultaneous knowledge of (σ'_p/P_a, σ'_{v0}/P_a, PL, LL, w_n) from 141 sites worldwide. Data points with OCR > 20 (possibly fissured clays) are excluded. Table 2 shows the statistics for (σ'_p/P_a, σ'_{v0}/P_a, PL, LL, w_n) and OCR. In the following, (Y_1, Y_2, …, Y_5) are defined as

$$\begin{aligned} Y_1 &= \ln(\sigma'_p/P_a)\\ Y_2 &= \ln(\sigma'_{v0}/P_a)\\ Y_3 &= PL\\ Y_4 &= LL\\ Y_5 &= w_n \end{aligned} \tag{1}$$

Table 2
Statistics for (σ'_p/P_a, σ'_{v0}/P_a, PL, LL, w_n) & OCR for 1217 data points in CLAY/10/7490.

Clay parameter	Random variable	Mean	Standard deviation	Min	Max
σ'_p/P_a	–	2.48	4.27	0.08	48.49
$\ln(\sigma'_p/P_a)$	Y_1	0.26	1.04	−2.55	3.88
σ'_{v0}/P_a	–	1.03	1.16	0.03	14.96
$\ln(\sigma'_{v0}/P_a)$	Y_2	−0.39	0.92	−3.60	2.71
PL	Y_3	27.3	11.1	0.6	76.5
LL	Y_4	64.4	30.5	20.3	236.4
w_n	Y_5	60.9	31.7	10.3	193.4
OCR	–	2.47	2.48	0.46[a]	20

[a] There are six data points with OCR significant < 1 (under-consolidated).

where PL, LL, and w_n are in percents, e.g., if w_n = 90%, take w_n = 90. Fig. 1 shows the histograms for (Y_1, Y_2, Y_3, Y_4, Y_5).

2.1. Multivariate Johnson probability density function

The 1217 data points are adopted to construct the *prior* multivariate PDF of (Y_1, Y_2, …, Y_5) for the subsequent Bayesian analysis. It is common for real data points to follow non-normal distributions. This can be clearly seen in Fig. 1, where most histograms exhibit a certain degree of asymmetry, which is not consistent with the normal model. A practical approach for constructing the multivariate PDF is the translation approach (Liu and Der Kiureghian, 1986; Li et al., 2012): the non-normal data (Y_1, Y_2, …, Y_5) are first mapped into standard normal data (X_1, X_2, …, X_5). Then, (X_1, X_2, …, X_5) are *assumed* to follow the multivariate standard normal distribution.

2.2. Mapping (Y_1, Y_2, …, Y_5) into (X_1, X_2, …, X_5)

The first step for the translation approach is to map (Y_1, Y_2, …, Y_5) into (X_1, X_2, …, X_5). Ching and Phoon (2014b) and Ching et al. (2014, 2017b) have shown that the Johnson system of distributions (Phoon and Ching, 2013) is effective in modeling various histograms of soil parameters. We cover the Johnson system of distributions below, starting with a well known member: the shifted lognormal distribution. The lognormal distribution has zero as its lower bound. The shifted lognormal distribution generalizes the lognormal distribution to account for non-zero lower bounds. If Y is shifted lognormal, the relationship between X and Y is

$$\frac{X-b_X}{a_X} = \ln\left(\frac{Y-b_Y}{a_Y}\right) \tag{2}$$

where X is standard normal; a_X, b_X, a_Y, and b_Y are the parameters for the shifted lognormal distribution. The parameter b_Y is the lower bound of